COMPUTATIONAL ANALYSIS OF PHOTON SAIL PARAMETERS

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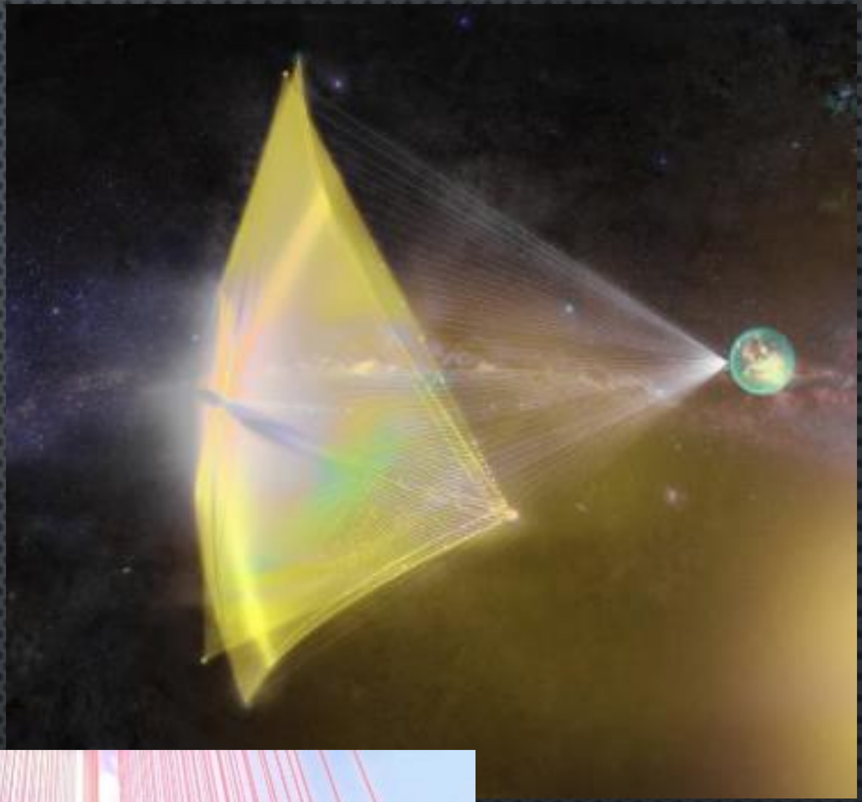
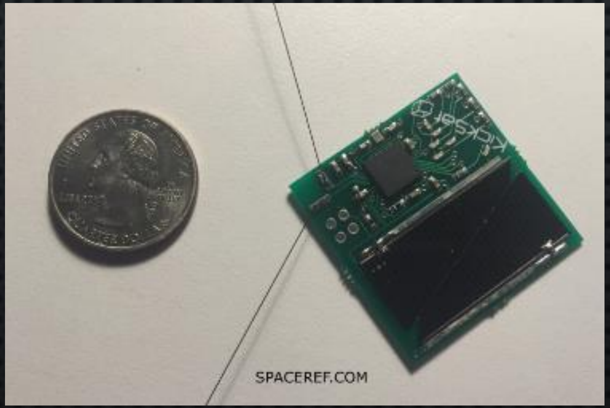
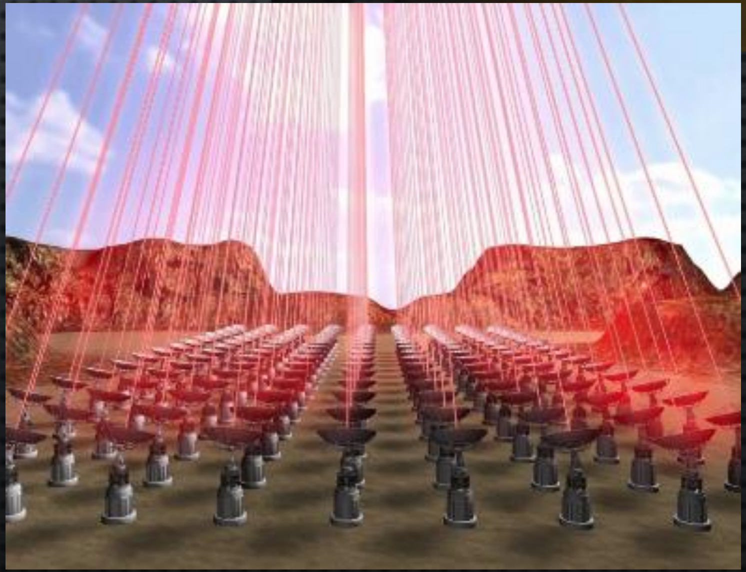
PROJECT SUMMARY

- Inspired by *Breakthrough Starshot Initiative*
- Computational Model in *Python*
- Objective: Determine most efficient means of maximizing velocity



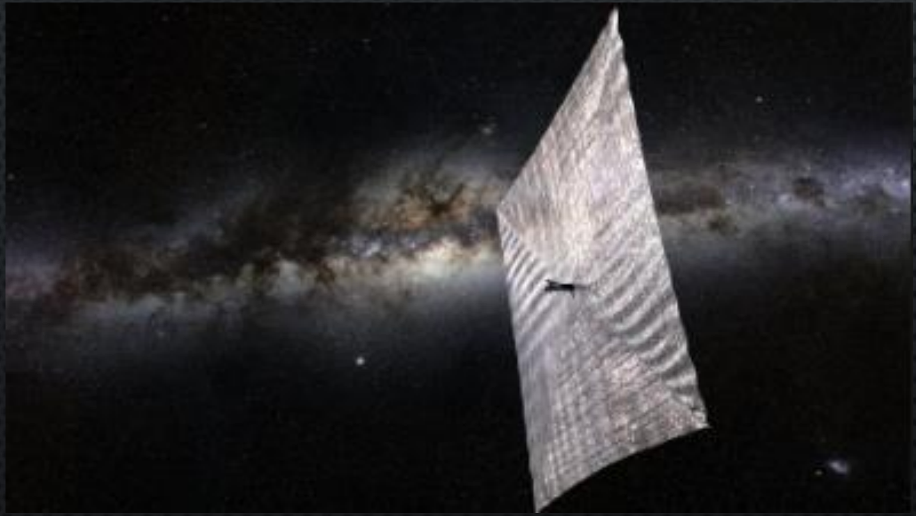




BACKGROUND – SPACE TRAVEL

- Rockets are expensive, ”slow” peak velocities, time consuming
- Roughly $400 billion to send a manned vessel to Mars (only about 40 million miles away), several months to get there
- We want to go further – rockets remain prisoner to Newton’s 2nd Law
- *Juno* spacecraft – fastest ever only at 45 mi/s (0.025 % *c*) – 17,000 yrs. to Alpha Centauri
- We need to go faster for cheaper

BACKGROUND – PHOTON SAILS

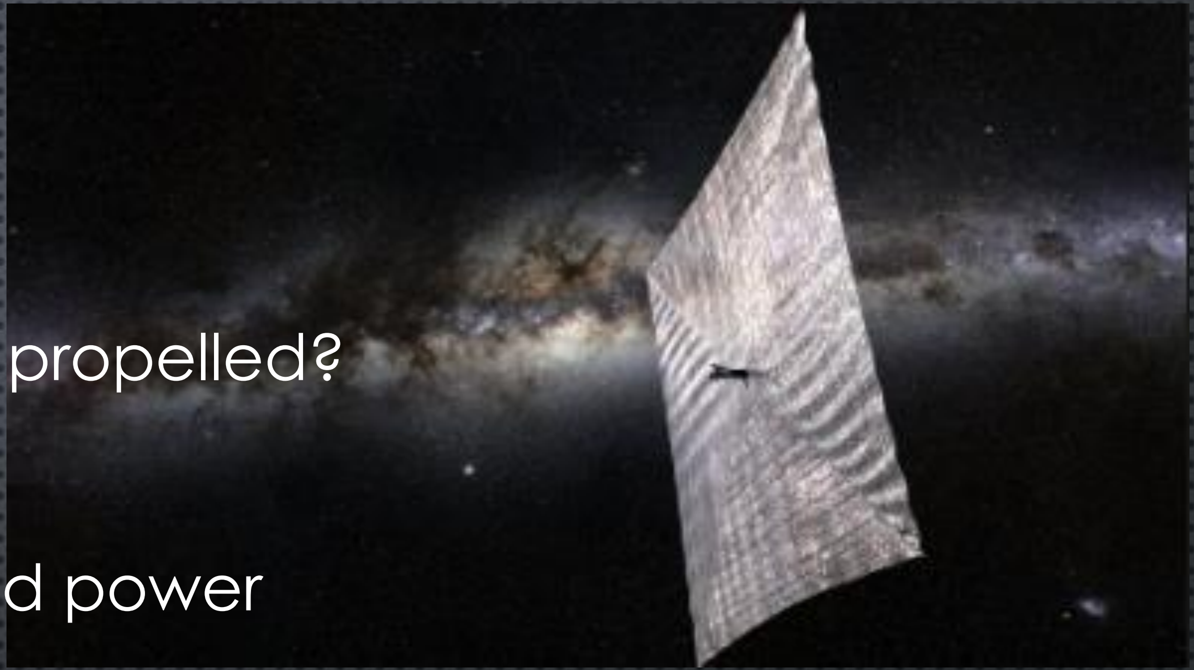


- Alternative means of space travel
- Originally proposed by Carl Sagan
- Large, thin, durable, reflective material used
- Exploits the momentum carried by photons (light particles)
- Solar sails inefficient, so we provide a stronger light source (Laser)

MY MODEL

- 1-Dimensional Simulation in Python
- Determine the factors that would affect Photon Sail's propulsion
- Derive an equation for the instantaneous acceleration of the craft
- Run trials by changing different parameters in order to find most efficient means of reaching maximum velocity

THEORY



- What will affect how fast the sail is propelled?
  - Sail’s surface area
  - Laser’s wavelength, size, and power
  - Instantaneous distance and velocity of the craft
  - Mass of the craft and its payload
  - Sail material’s albedo
  - Radiation Pressure from the Sun
  - Doppler Red-shift at relativistic speeds

THE INSTANTANEOUS ACCELERATION EQUATION

$$\vec{a}_i = \kappa * \frac{SA}{\pi * m_{rel} * c} * \frac{\sqrt{1 - \frac{\vec{v}_{i-1} + \vec{a}_{i-1} * t_{step}}{c}}}{\sqrt{1 + \frac{\vec{v}_{i-1} + \vec{a}_{i-1} * t_{step}}{c}}} * \left(\frac{P_0}{\left(\left\{\left[d_{i-1} + \vec{v}_{i-1} * t_{step} + \frac{1}{2}\vec{a}_{i-1} * (t_{step})^2\right] * \tan\left(1.22\,\frac{\lambda_0}{D}\right)\right\} + \frac{D}{2}\right)^2} + \frac{L_{sun}}{4\,\left(\left[d_{i-1} + \vec{v}_{i-1} * t_{step} + \frac{1}{2}\vec{a}_{i-1} * (t_{step})^2\right] + 1\,AU\right)^2} \right)$$

- Derived using basic version of Newton's 2nd Law: $\vec{a} = \frac{\vec{F}}{m}$
- Force all from Radiation Pressure $\vec{F}_{rad} = \frac{I * SA}{c}$

<u>Key:</u>

$\vec{a}_i$ = acceleration after time step i — κ = albedo correction factor — c = speed of light (m/s)

m_{rel} = relativistic mass correction — π = pi — $\vec{v}_i$ = velocity after time step i

t_{step} = time step or Δt — $1\,AU$ = one astronomical unit (m) — SA = sail's surface area

λ_0 = initial wavelength of laser — D = diameter of laser collective aperture

L_{Sun} = luminosity of the Sun — P_0 = output power of laser — d_i = position after time step i

- Intensity constantly changing, thus acceleration is constantly changing
- Laser's Wavelength
- Laser's Power
- Laser Array Diameter
- Doppler Red shift

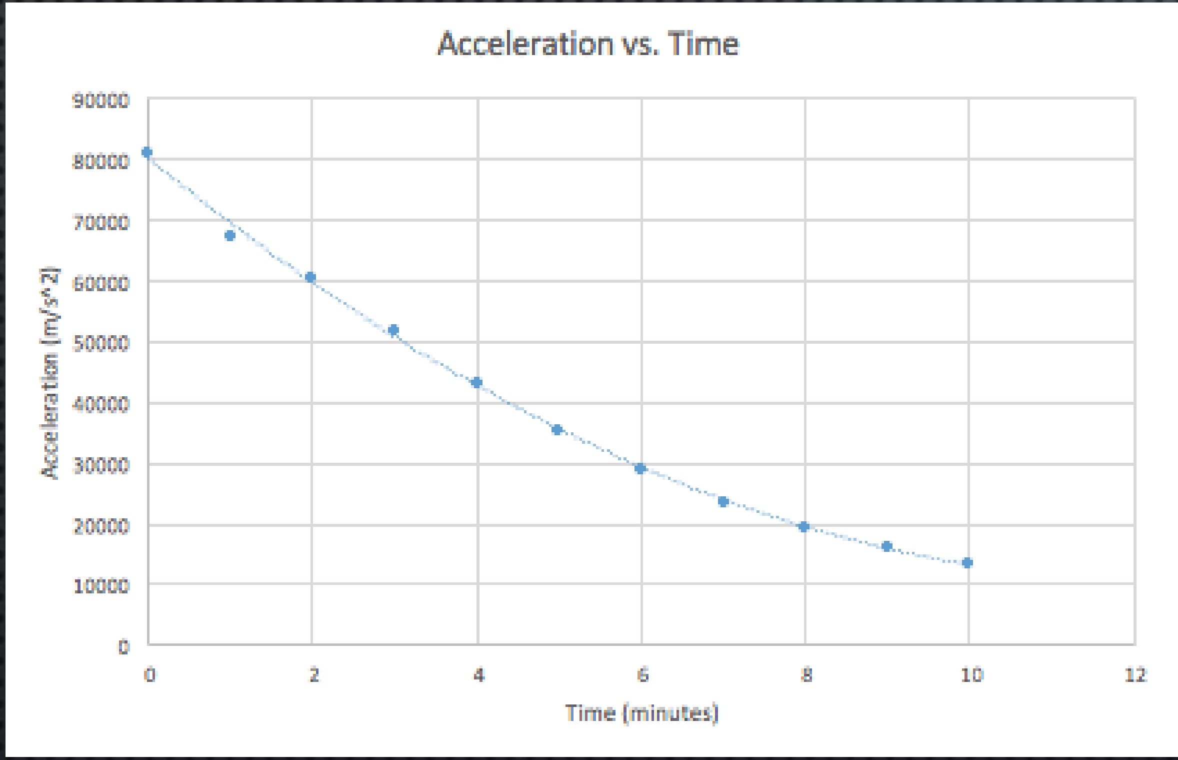
IMPLEMENTING THE CODE

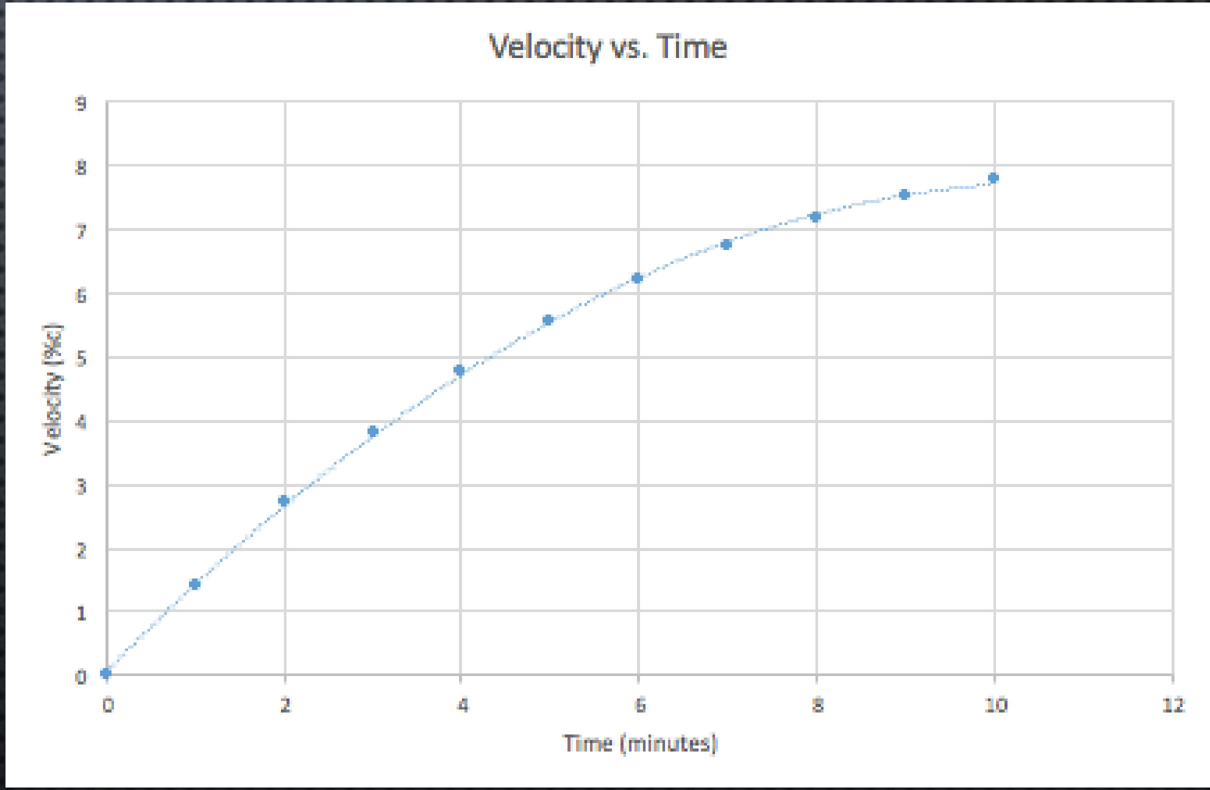
```
def newAcceleration(prevDistance, prevVelocity, prevAcceleration, sailArea,
                    totalRelativisticMass, laserPower, laserWavelength,
                    laserDiameter):
    rayleighAngle = (1.22*laserWavelength)/(laserDiameter)
    beamArea = ((m.tan(rayleighAngle))*(prevDistance)+laserDiameter/2)**2
    intensity = laserPower/beamArea
    redshiftCorrection = (m.sqrt((1-prevVelocity/c)/(1+prevVelocity/c)))
    af = (redshiftCorrection)*(sailArea/(pi*totalRelativisticMass*c))*(((
    intensity)) + (Lsun)/(4*((prevDistance+xls)**2)))
    return af
```

- Acceleration equation: foundation of the program
- Problem: Instantaneous acceleration depended on the instantaneous distance and velocity, and the instantaneous distance and velocity depended on the instantaneous acceleration
- Solution: Time-steps (the ***i*** and ***i-1*** subscripts) – *new* ***a*** found by *previous* ***d*** & ***v***

$$\vec{a}_i = \kappa * \frac{SA}{\pi * m_{rel} * c} * \frac{\sqrt{1 - \frac{\vec{v}_{i-1} + \vec{a}_{i-1} * t_{step}}{c}}}{\sqrt{1 + \frac{\vec{v}_{i-1} + \vec{a}_{i-1} * t_{step}}{c}}} * \left(\frac{P_0}{\left(\left\{\left[d_{i-1} + \vec{v}_{i-1} * t_{step} + \frac{1}{2}\vec{a}_{i-1} * (t_{step})^2\right] * \tan\left(1.22\,\frac{\lambda_0}{D}\right)\right\} + \frac{D}{2}\right)^2} + \frac{L_{sun}}{4\left(\left[d_{i-1} + \vec{v}_{i-1} * t_{step} + \frac{1}{2}\vec{a}_{i-1} * (t_{step})^2\right] + 1\,AU\right)^2} \right)$$

BASIC TRENDS OF CRAFT'S JOURNEY





DATA AND RESULTS

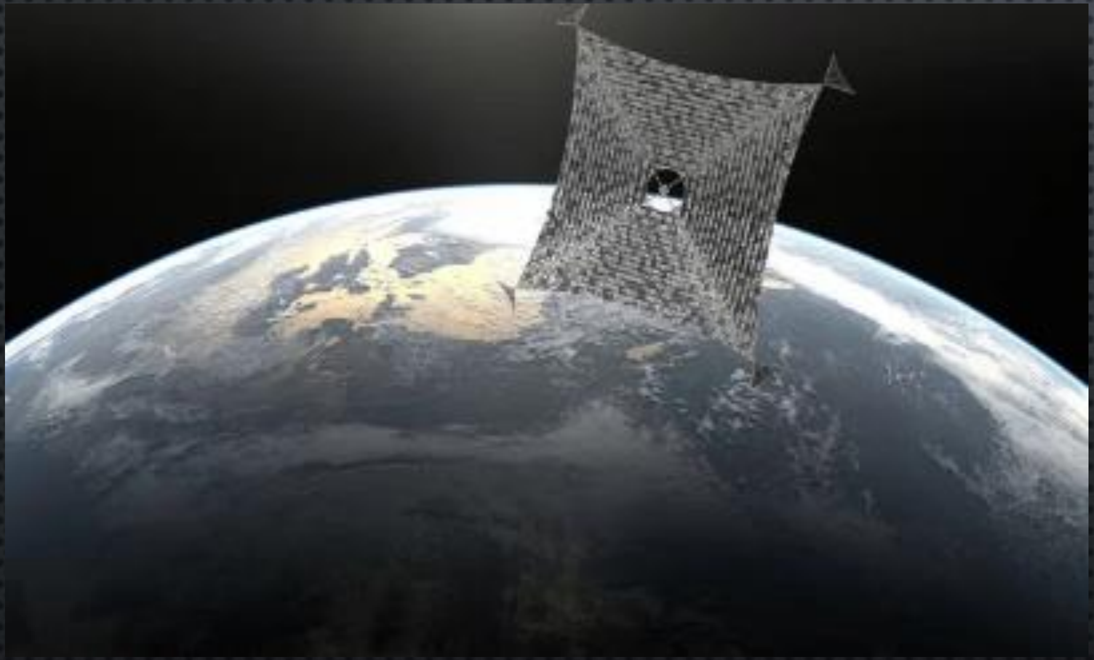
Trial	Laser Output Wattage (GW)	Laser Wavelength (nm)	Sail Dimensions (m)	Laser Array Diameter (m)	Total Mass (kg)	Max Velocity (%c)	Time to Proxima Centauri (yr)
*BSI**	100	700 (red light)	4 x 4	100	0.003	**1.7**	248.6
1	100	700 (red light)	100 x 100	100	0.0594	**9.32**	45.3
2	100	550 (green light)	100 x 100	100	0.0594	**10.48**	40.3
3	100	400 (violet light)	100 x 100	100	0.0594	**12.21**	34.5
4	100	300 (ultra-violet light)	100 x 100	100	0.0594	**14.01**	30.1
5	250	550 (green light)	100 x 100	100	0.0594	**16.22**	26
6	500	550 (green light)	100 x 100	100	0.0594	**22.39**	18.8
7	750	550 (green light)	100 x 100	100	0.0594	**26.91**	15.7
8	1000	550 (green light)	100 x 100	100	0.0594	**30.59**	13.4
9	100	550 (green light)	4 x 4	100	0.003	**1.92**	220.2
10	100	550 (green light)	10 x 10	100	0.0061	**3.36**	125.7
11	100	550 (green light)	50 x 50	100	0.00281	**7.69**	54.8
12	100	550 (green light)	100 x 100	100	0.0594	**10.48**	40.3
13	100	550 (green light)	200 x 200	100	0.1346	**13.76**	30.7
9	100	550 (green light)	100 x 100	10	0.0594	**8.98**	47
10	100	550 (green light)	100 x 100	50	0.0594	**10.43**	40.5
11	100	550 (green light)	100 x 100	100	0.0594	**10.48**	40.3
12	100	550 (green light)	100 x 100	200	0.0594	**10.49**	40.2
13	100	550 (green light)	100 x 100	350	0.0594	**10.49**	40.2
14	100	550 (green light)	100 x 100	100	0.0594	**10.48**	40.3
15	100	550 (green light)	100 x 100	100	0.0634	**10.15**	41.6
16	100	550 (green light)	100 x 100	100	0.0684	**9.79**	43.1
17	100	550 (green light)	100 x 100	100	0.1584	**6.51**	64.8
18	100	550 (green light)	100 x 100	100	~1	**2.63**	160.7
*19***	100	550 (green light)	100 x 100	100	0.0634	**10.16**	42.6

CONCLUSION

- *Max Velocities* are reached when:
  - ➢ Decreasing Laser's wavelength
  - ➢ Increasing Laser's output power
  - ➢ Laser Array diameter around 100 meters
  - ➢ Increasing Sail's surface area (below threshold)
  - ➢ Minimizing craft's overall mass

- *Only ideal for gram-scale crafts*
- Human-based interstellar travel with photon sails unrealistic

QUESTIONS?



MORE TO CONSIDER:

- Universe is 3-D –complex system entails extreme accuracy needed
- Keeping the laser pointed at the craft for several hours / days
- Steering the craft
- Other bodies within the solar system (big and small)
- Collisions with particles at high speeds
- Durability, heat capacity, protecting the microchip
- Collecting data during fly-bys at relativistic speeds
- Financial burden – research, materials, powering the laser, placing crafts in HAO
- Ground-based laser would experience atmospheric interference