CSAAPT
Loyola University

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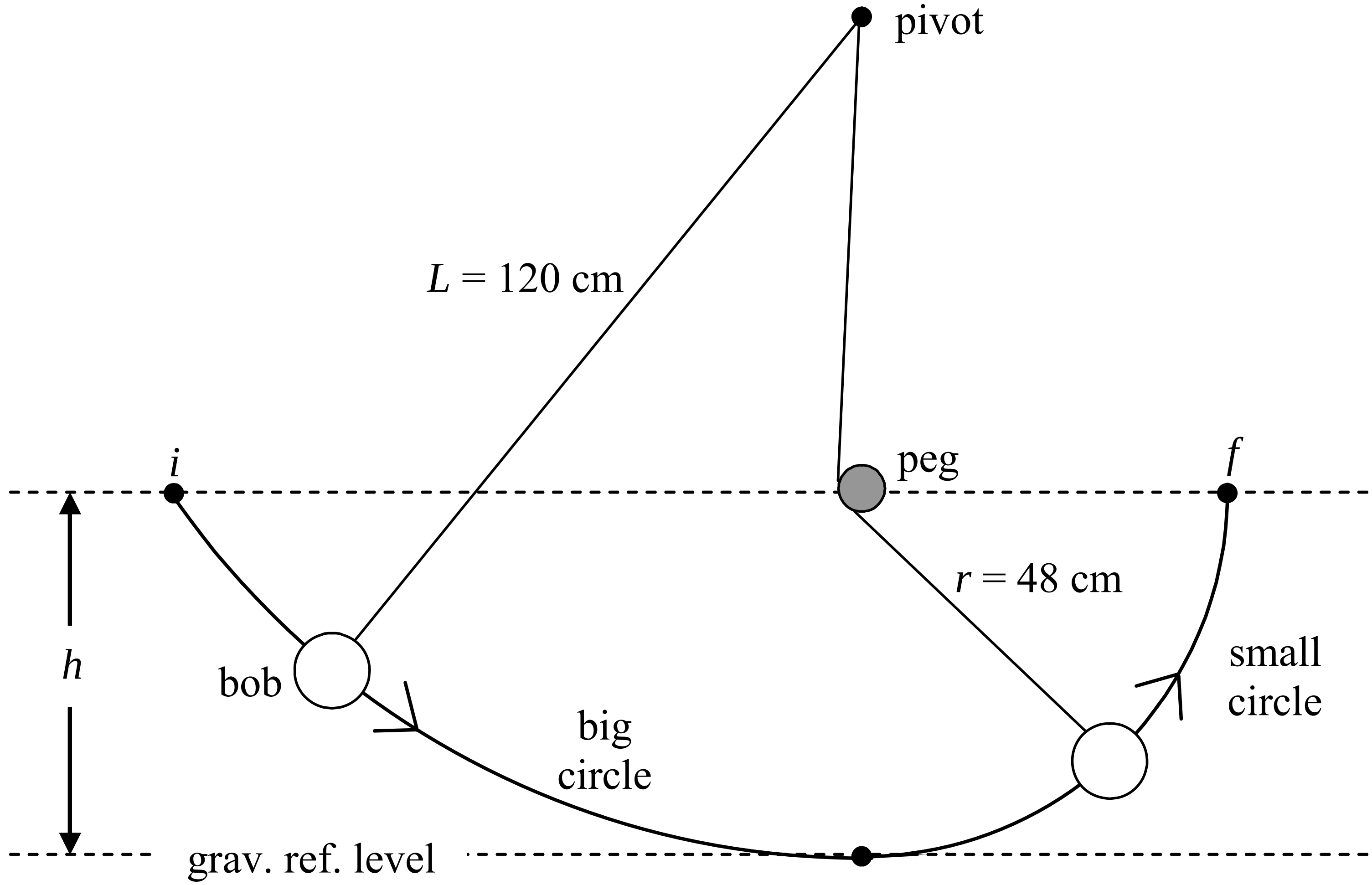
Demonstration: The Loop-the-Loop Pendulum

Carl E. Mungan

Physics Department *U.S. Naval Academy*



Initial point i and final point f must be at the same height, for example $h = r$ here.

PROBLEM STATEMENT

At what minimum initial height h must the bob be released so that the string smoothly wraps around the peg?

PLEASE ANSWER AS IF YOU WERE A STUDENT!

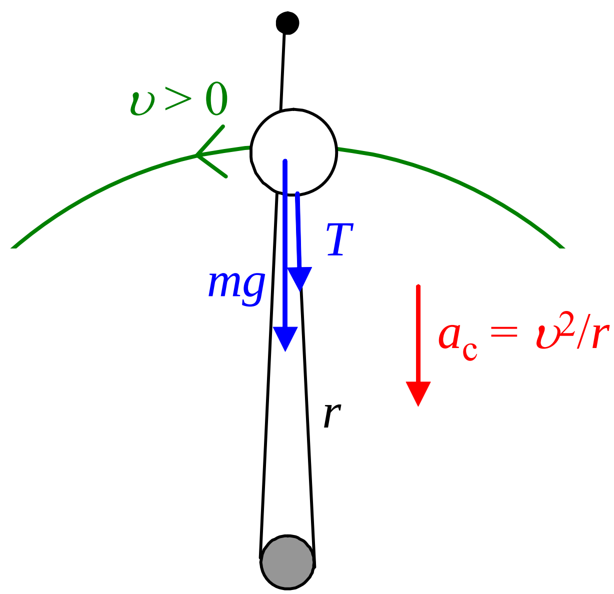
Let's try $h = 2r$.

We need $\upsilon > 0$ at the top of the small circle!

THEORY: combined energy & force analysis

First we need to draw a FBD even to do the *energy* analysis.
How come?

Because E_{mech} is conserved if and only if $W_{\text{nc}} = 0$.



$W_{\text{nc}} = W_T = 0$ since the tension and bob's motion are perpendicular

ENERGY ANALYSIS: $E_{\text{mech}} = K + U_{\text{g}} = \text{constant}$

$$E_{\text{mech,initial}} = 0 + mgh \quad \text{(released from rest at } h\text{)}$$

$$= E_{\text{mech,final}} = \frac{1}{2}m\upsilon^2 + mg2r \quad \text{(at top of small circle)}$$

$$\therefore \upsilon^2 = g\left(2h - 4r\right)$$

this equation relates υ to h

FORCE ANALYSIS: $\Sigma F = ma$

$$T + mg = m\frac{\upsilon^2}{r} \quad \text{(at top of small circle)}$$

If the bob *just* makes it around, what is happening to the string?

It's on the verge of collapsing so $T \to 0$.

(Similar to the vertically spun water bucket when $N \to 0$.)

so $mg = m\dfrac{\upsilon^2}{r}$ but we had $\upsilon^2 = g\left(2h - 4r\right)$

$$\therefore r = 2h - 4r \quad \Rightarrow \quad \boxed{h = \frac{5}{2}r}$$

Let's try $h = 2.5r$ which here happens to equal L.

(This demo works much better than the marble loop-the-loop because the marble has too much friction.)