What is entropic dissipation?

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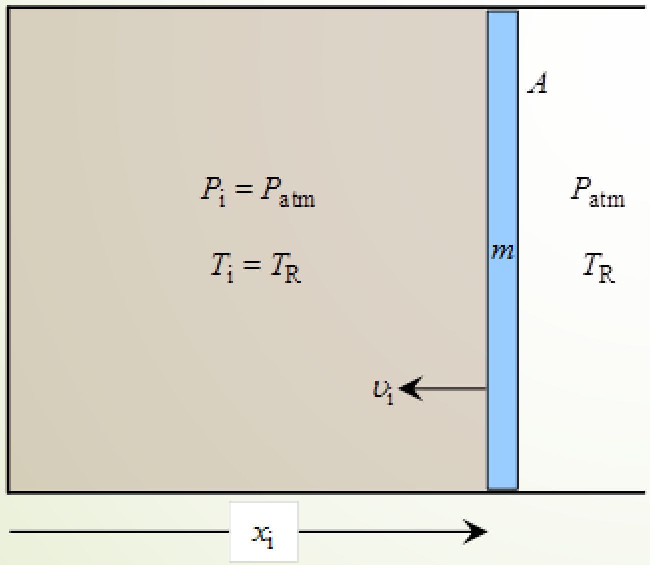
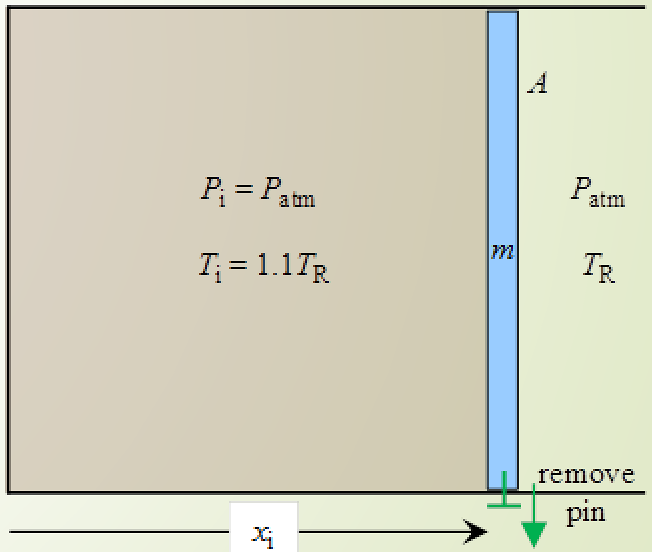
Consider a *monatomic* ideal gas enclosed in a *diabatic* cylinder by a *frictionless* sliding piston. Find $x(t)$ in both cases.

Case A: The piston has mass ($m > 0$), starts out at room temperature, and is given an impulsive inward kick.



Case B: The piston is massless ($m = 0$), starts out 10% above room temperature, and a holding pin is suddenly removed.



Are those two processes thermodynamically reversible or not?

They are both irreversible because the cylinder walls and piston are thermally conducting. Whenever heat is transferred between two objects (here the ideal gas and the atmosphere) that differ in temperature by a non-infinitesimal amount, the entropy of the universe increases.

Nevertheless these two processes are quasistatic if what condition is true?

The gas is in quasi-equilibrium (so that it can be described by one overall pressure and temperature) if the piston always moves much slower than the gas atoms, i.e., if the maximum speed of the piston is much smaller than the speed of sound. Then there will never be a partial vacuum or compression behind the piston.

Okay, we can reasonably assume the processes are irreversible but quasistatic. Now what two principles (equations) can we use to solve for the piston's motion?

Newton's second law (N2L): $PA - P_{\text{atm}}A = ma \quad \Rightarrow \quad \boxed{\frac{nRT}{x} - P_{\text{atm}}A = m\dot{\upsilon}}$

first law of thermo (T1L): $dU = dQ_{\text{in}} + dW_{\text{on}} \quad \Rightarrow \quad \frac{3}{2}nRdT = -knR(T - T_{\text{R}})dt - \frac{nRT}{V}dV$

(where k is the thermal conductance) $\quad \Rightarrow \quad \boxed{\frac{3}{2}\dot{T} = -k(T - T_{\text{R}}) - \frac{T}{x}\upsilon}$

So we have two coupled DEs (in the boxes) for $x(t)$ and $T(t)$ to solve.

Euler-Cromer method in a spreadsheet for case A:

start the iteration by defining

$$T = T_i$$
$$\upsilon = \upsilon_i$$
$$x = x_i$$
$$t = 0$$

on the next spreadsheet row use Eqs. (2) and (4) to compute

$$T_{new} = T + \dot{T}dt$$
$$\upsilon_{new} = \upsilon + \dot{\upsilon}dt$$
$$x_{new} = x + \upsilon_{new}\,dt$$
$$t_{new} = t + dt$$

redefine

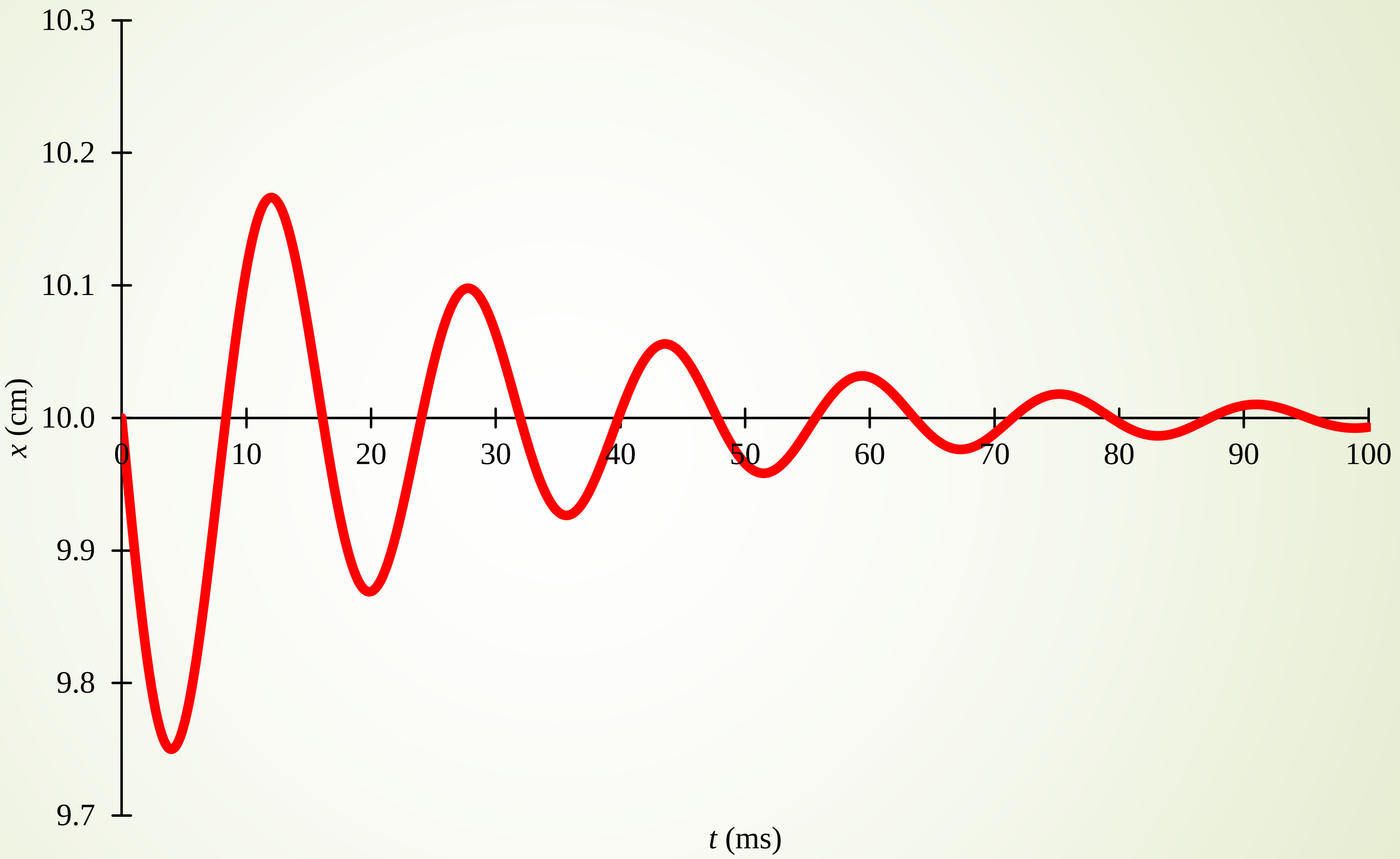
$$T = T_{new}$$
$$\upsilon = \upsilon_{new}$$
$$x = x_{new}$$
$$t = t_{new}$$

repeat

Cromer's modification to Euler uses υ_{new} rather than υ to find x_{new}

Eqs. (2) and (4) are boxed on the previous slide

Result for case A:



Thus the motion of the piston consists of underdamped oscillations.

Unlike the numerical solution required in case A, an analytic solution exists for case B. (See if you can guess its form before I get to it!)

$$\text{N2L} \quad F_{\text{net}} = ma = 0 \quad \text{since massless}$$

$$\therefore \text{gas pressure} = \text{atmospheric always}$$

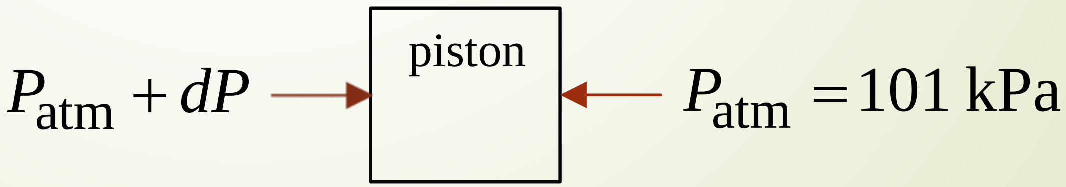
$$\Rightarrow T(x) = P_{\text{atm}} Ax / nR$$

But if the pressures balance on the piston, how does it start moving?

<u>Comment #1</u> $F_{\text{net}} = 0$ does NOT mean $a = 0$.

In fact $a = F_{\text{net}} / m = 0/0$ can have any value!

<u>Comment #2</u> Two quantities are numerically equal even if they differ infinitesimally:



The numerical value of $P_{\text{atm}}+dP$ is still 101 kPa!

$$\text{N2L} \Rightarrow T(x) = P_{\text{atm}} Ax / nR$$

Substitute that result into T1L in three places:

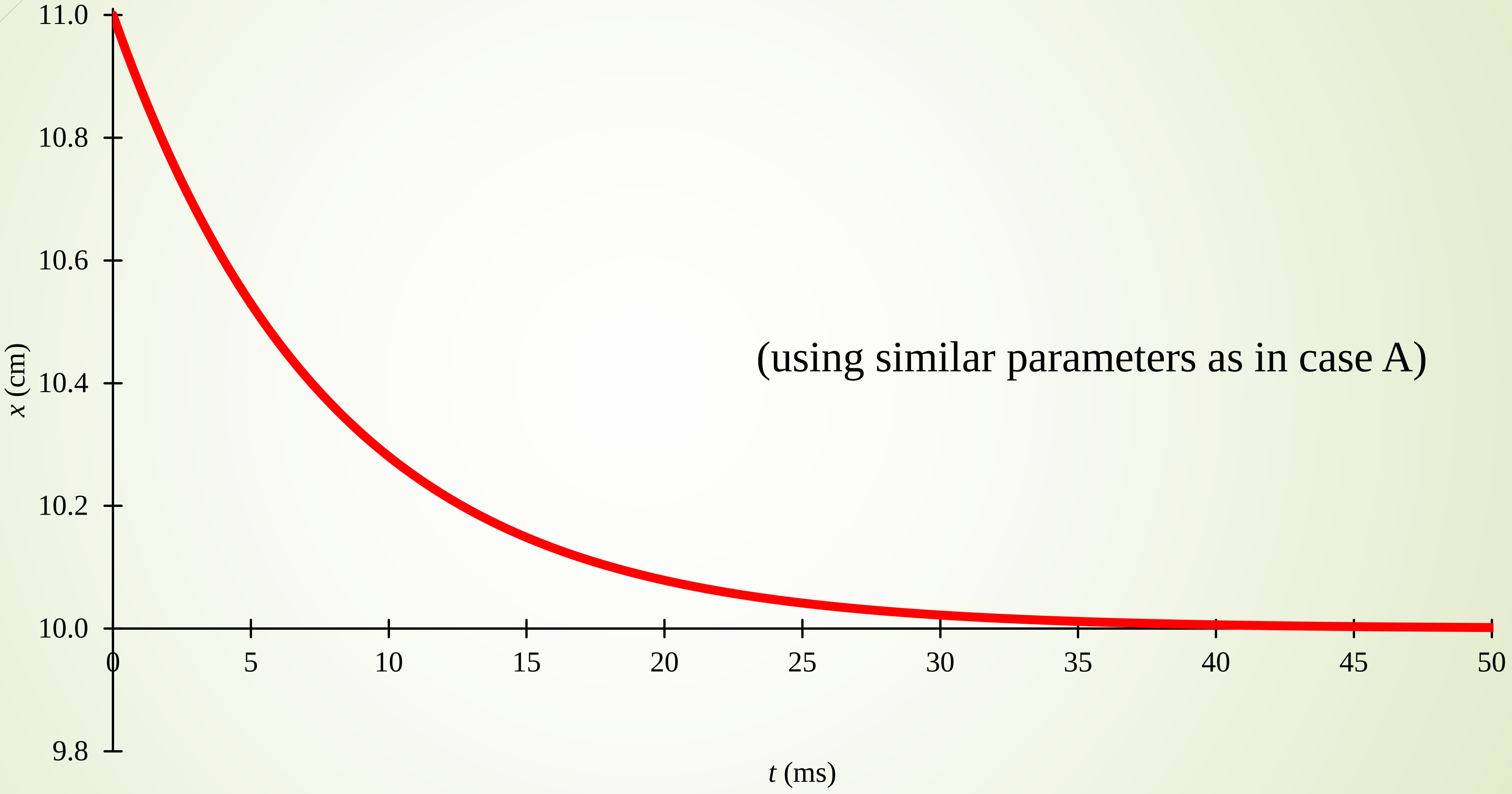
$$\frac{3}{2} dT = -k(T - T_{\text{R}})dt - \frac{T}{x} dx$$

to obtain a separable DE with solution:

$$\boxed{x(t) = (x_{\text{i}} - x_{\text{f}})e^{-0.4kt} + x_{\text{f}}}$$

Do you recognize this solution compared to that of case A?

Solution for case B:



(using similar parameters as in case A)

Now the motion of the piston is overdamped!

PUZZLE: WHY IS THE MOTION DAMPED?

The piston slides frictionlessly, an ideal gas has no viscosity or turbulence, and atmospheric drag was not included… so what causes the loss?

Well, the process is irreversible and so the entropy of the universe monotonically increases (regardless of whether the piston moves inward or outward, and of whether the gas is colder or hotter than the atmosphere).

But entropy cannot increase forever! It must reach a maximum which defines the final equilibrium state. In other words, there is loss driven by entropy.

THAT IS CALLED "ENTROPIC DISSIPATION."

A flowchart coupling entropy to force explains the dissipation:

ENTROPY↔HEAT ↔TEMPERATURE ↔PRESSURE ↔FORCE

Interestingly the two starting equations were those for force (N2L) and energy (T1L).

The second law of thermo (T2L) for the entropy emerges naturally from the dynamics without having been explicitly inserted.