Video Analysis of an Unrolling Mat Using Tracker

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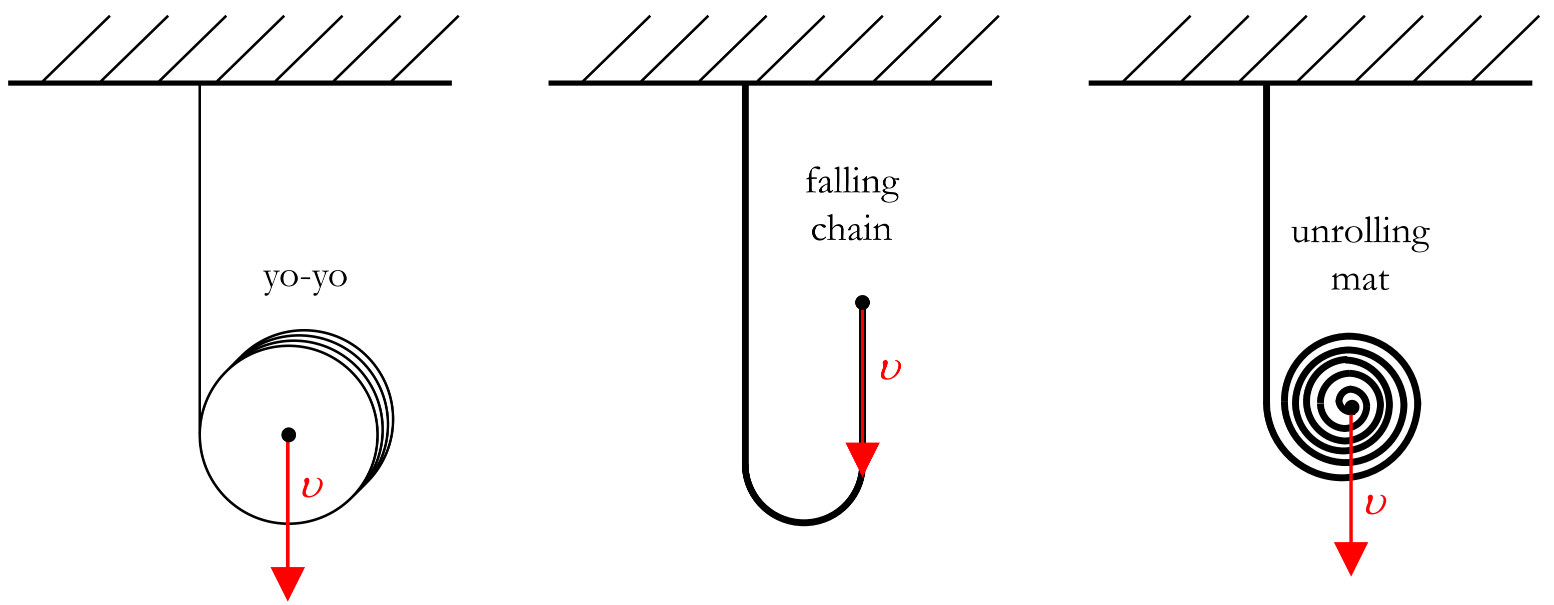
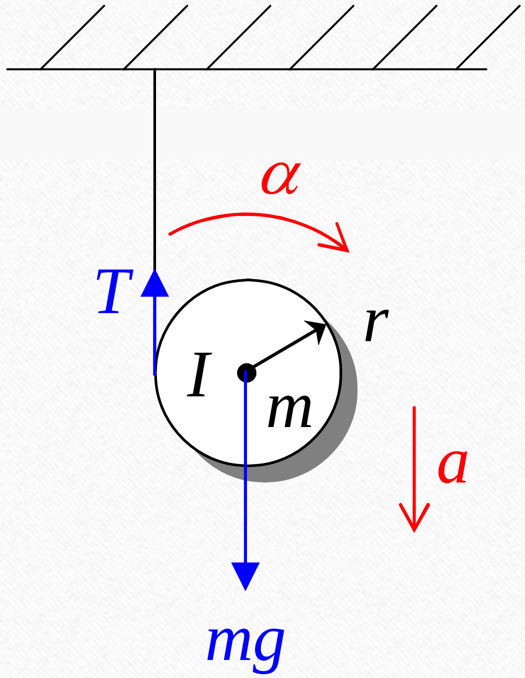
Which of these systems conserve mechanical energy during their descent?



YO-YO



Newton's second law for translations and rotations:

$$mg - T = ma \qquad Tr = I\frac{a}{r}$$

eliminate T to get $a = \dfrac{g}{1 + I/mr^2}$

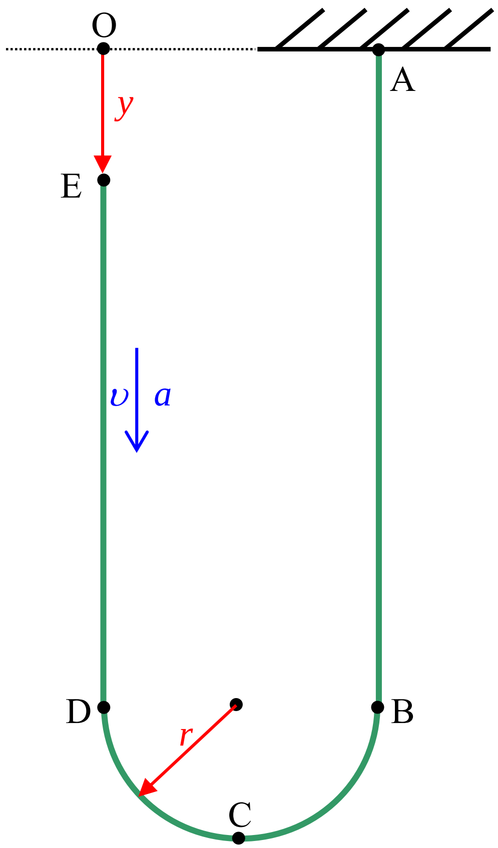
but $\upsilon^2 = 2ah = (r\omega)^2$

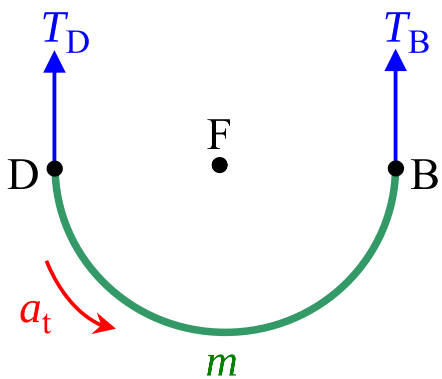
so that $K = \frac{1}{2}m\upsilon^2 + \frac{1}{2}I\omega^2 = mgh = U$

Therefore we deduce from Newton's second law and kinematics for constant acceleration that mechanical energy is conserved during the fall of the yo-yo.

This deduction is consistent with what we expect intuitively. The string is massless, air drag is neglected, and the point of attachment of the string to the ceiling does not move. Thus there is no mechanism for the yo-yo to lose mechanical energy.

FALLING CHAIN of linear mass density λ



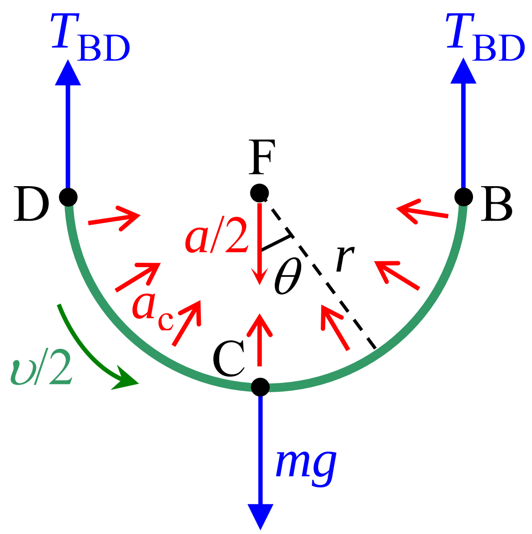


$$T_B - T_D = m a_t$$

$$m \approx 0 \quad \Rightarrow \quad T_B = T_D \equiv T_{BD}$$

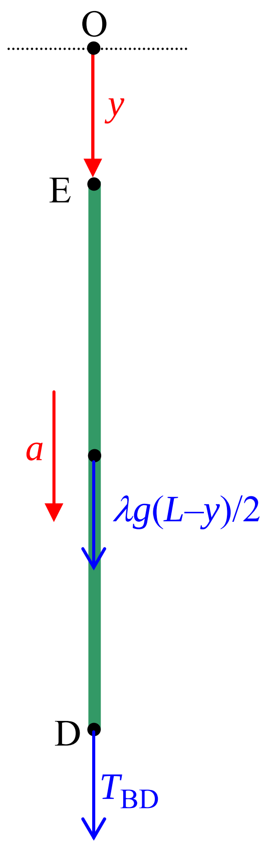
like tension around an ideal pulley

Point D falls with speed υ and point B with zero speed.
Point F falls with their average speed $\upsilon/2$.
Jump into an inertial reference frame instantaneously comoving with F.



$$2T_{BD} - \lambda\pi r g = \lambda\pi r\left(\frac{\upsilon^2}{4r}\frac{2}{\pi} - \frac{a}{2}\right)$$

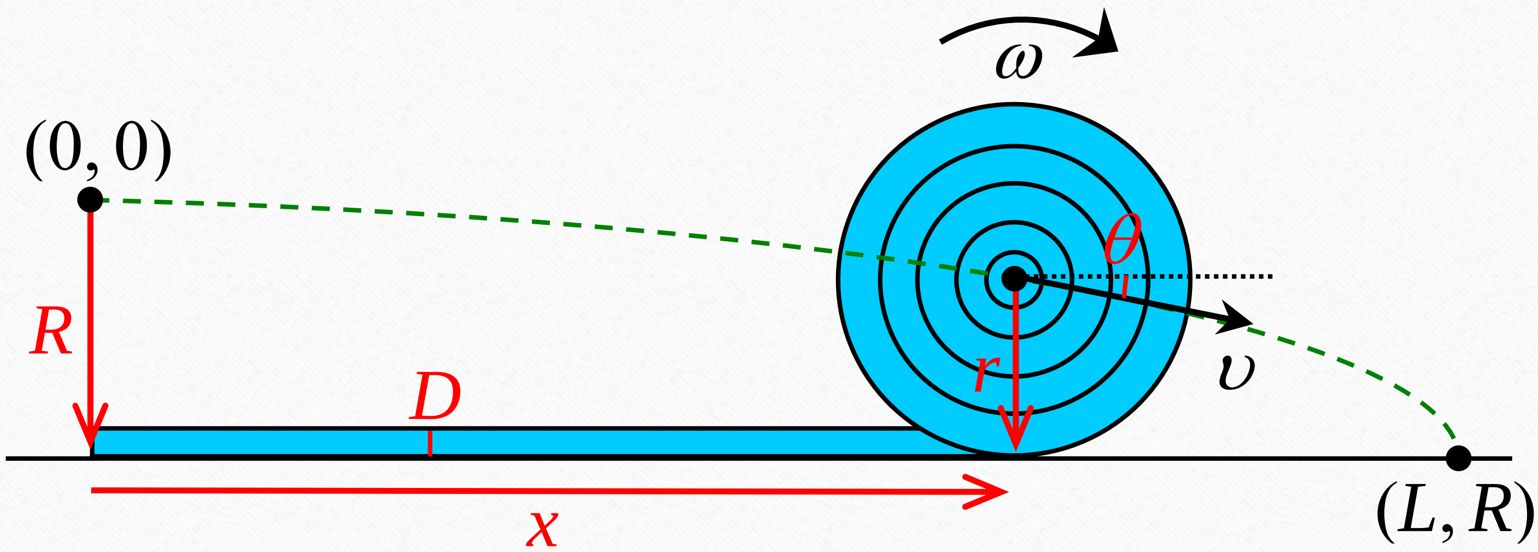
$$\text{but } r \approx 0 \quad \Rightarrow \quad T_{BD} = \tfrac{1}{4}\lambda\upsilon^2$$



Substituting in the expression for T_{BD} leads to the surprising result that the mechanical energy of the chain is overall constant. The ejected links produce a “rocket” thrust of the remaining falling segment such that no KE is lost.

Thus kinetic energy gets “concentrated” in the falling segment.

UNROLLING MAT



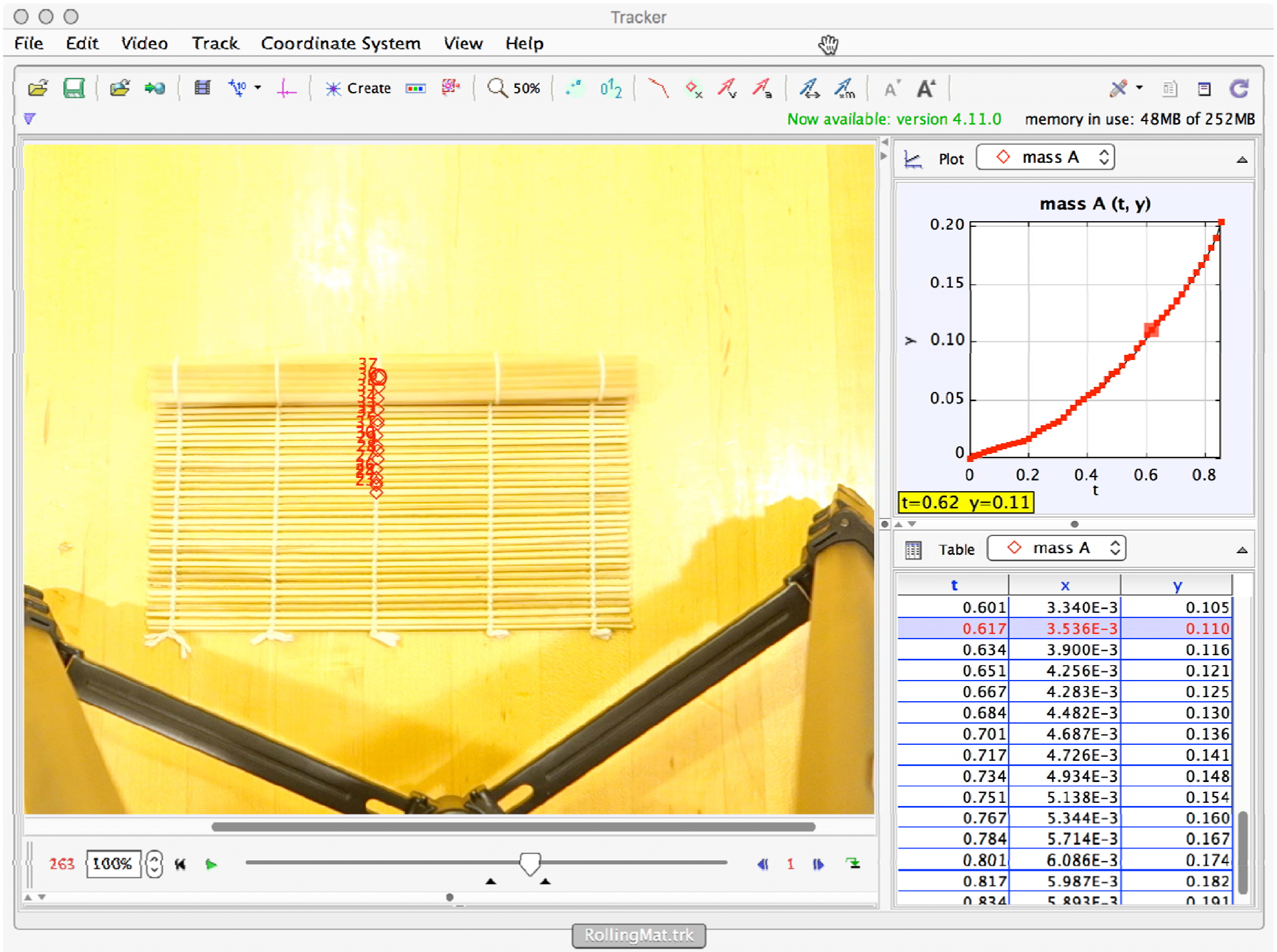
Theory due to Freeman in AJP 1946
assuming conservation of mechanical energy:

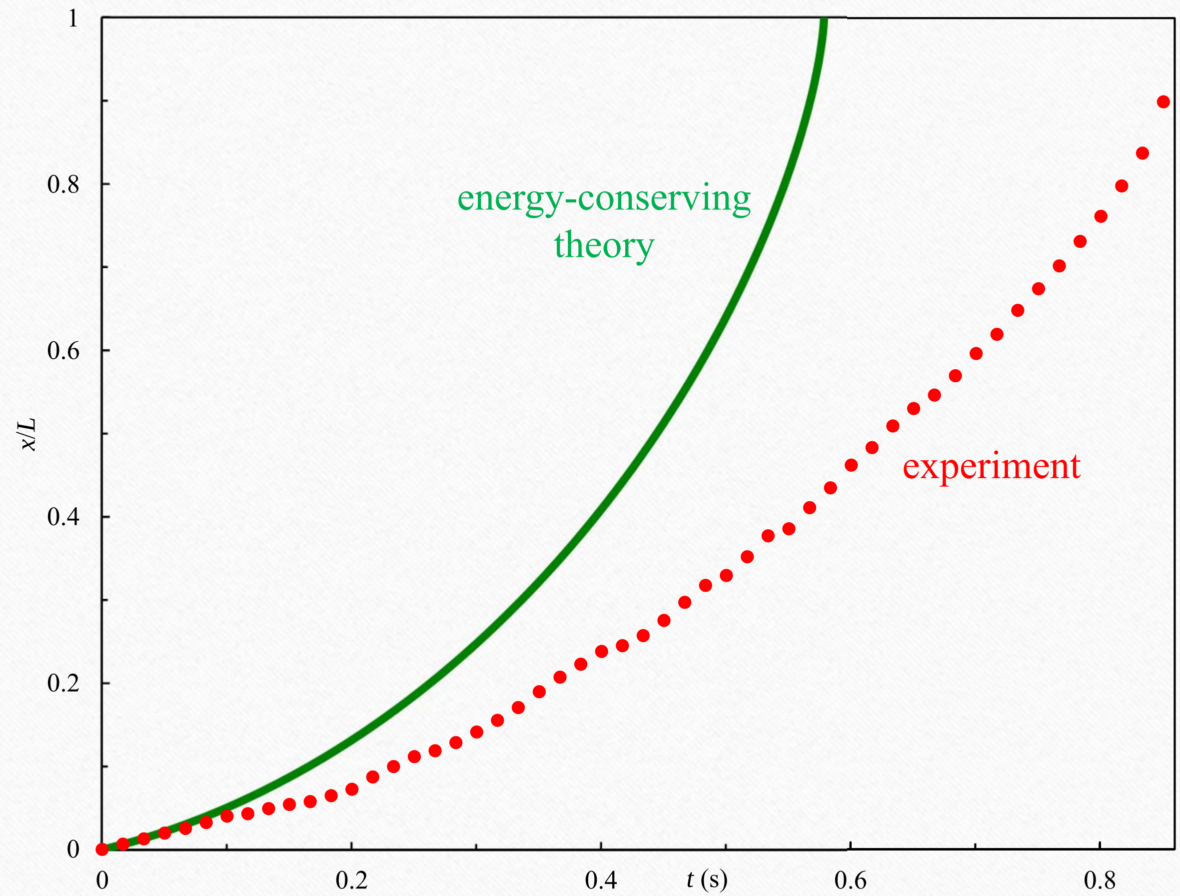
PE lost by roll is $U = MgR\left[1-(1-x/L)^{3/2}\right]$

final KE of roll is $K = \frac{M}{8}\left[\frac{d(x/L)}{dt}\right]^2\left[R^2 + 6L^2(1-x/L)\right]$

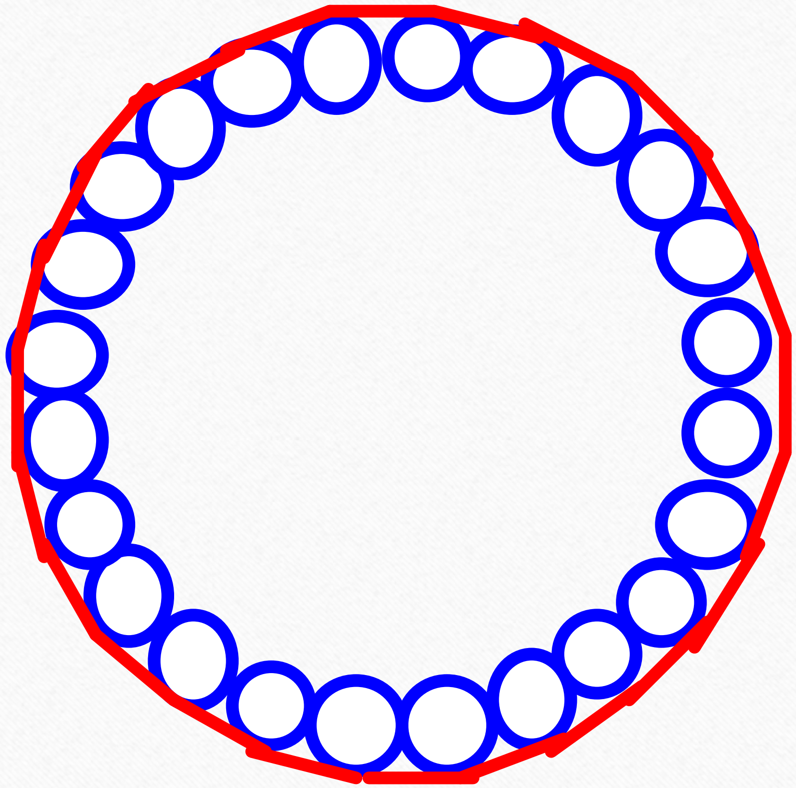
include initial KE to get $K - K_0 = U$

and integrate numerically to find x/L vs t


Tracker
File
Edit
Video
Track
Coordinate System
View
Help
Create
50%
Now available: version 4.11.0
memory in use: 48MB of 252MB
Plot
mass A
mass A (t, y)
0.20
0.15
0.10
0.05
0
y
0
0.2
0.4
0.6
0.8
t
t=0.62 y=0.11
Table
mass A
t x y
0.601 3.340E-3 0.105
0.617 3.536E-3 0.110
0.634 3.900E-3 0.116
0.651 4.256E-3 0.121
0.667 4.283E-3 0.125
0.684 4.482E-3 0.130
0.701 4.687E-3 0.136
0.717 4.726E-3 0.141
0.734 4.934E-3 0.148
0.751 5.138E-3 0.154
0.767 5.344E-3 0.160
0.784 5.714E-3 0.167
0.801 6.086E-3 0.174
0.817 5.987E-3 0.182
263
100%
1
RollingMat.trk



energy-conserving theory
experiment
x/L
t (s)
0
0.2
0.4
0.6
0.8
1
0
0.2
0.4
0.6
0.8

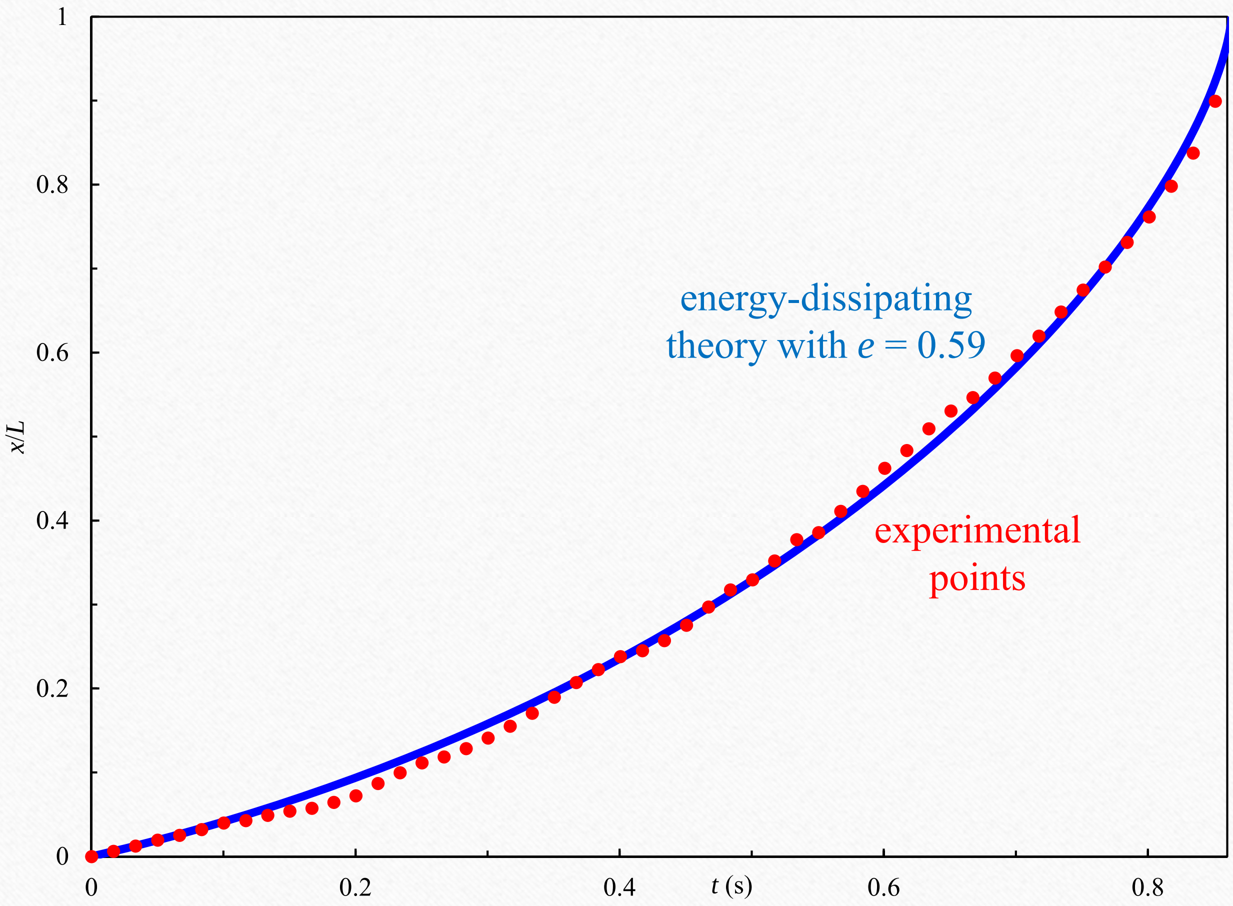


Connect adjacent outwardly radial "high points" of the bamboo rods by straight line segments, so that the rolled-up mat is a polygon.

The faces of a polygonal cylinder make inelastic collisions with the table, so we now get

$$K - K_0 = e^2 U$$

with coefficient of restitution *e*.



1
0.8
0.6
0.4
0.2
0
x/L
0
0.2
0.4
t (s)
0.6
0.8
energy-dissipating
theory with e = 0.59
experimental
points

CONCLUSIONS

● An ideal yo-yo conserves mechanical energy like a rigid wheel rolling down a ramp. The string tension like static friction does no work.

● An unfolding chain surprisingly also conserves mechanical energy. The KE lost by links turning the bottom corner is added to that of the remaining falling segment as a “rocket” thrust.

● However an unrolling mat cannot simultaneously be both ideally flexible and round in cross section. Hence there is dissipation of mechanical energy as it makes inelastic collisions with the tabletop. The fitted COR agrees with known wood-on-wood values of 0.4−0.6.

●Future work will study an unrolling window shade when there is no surface contact to cause energy dissipation.