Spectral Analysis Using Excel

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Overview

Calculating Fourier transform using Excel.

Spectral analysis of sound from a guitar.

Applications of Fourier Transform

Signal processing

Quantum mechanics: solving the time-dependent Schrödinger equation

Evolution of a Wave Packet in a 1D Potential

$$i\hbar\frac{\partial\psi}{\partial t} = H\psi$$

$$H = \frac{(\hbar k)^2}{2m} + V(x)$$

$$\psi(x,t) = e^{-i\frac{H}{\hbar}t}\,\psi(x,0)$$

$$\psi(x,\Delta t) \cong e^{-\frac{i}{\hbar}V(x)\frac{\Delta t}{2}} e^{-\frac{i\,(\hbar k)^2}{\hbar\ 2m}\Delta t} e^{-\frac{i}{\hbar}V(x)\frac{\Delta t}{2}} \psi(x,0)$$

Evolution of a Wave Packet in a 1D Potential

$$\psi(x,\Delta t) \cong e^{-\frac{i}{\hbar}V(x)\frac{\Delta t}{2}} e^{-\frac{i(\hbar k)^2}{\hbar\ 2m}\Delta t} e^{-\frac{i}{\hbar}V(x)\frac{\Delta t}{2}} \psi(x,0)$$

Diagonal in real space $\psi_1(x,\Delta t) = e^{-\frac{i}{\hbar}V(x)\frac{\Delta t}{2}}\psi(x,0)$

Diagonal in *k* space $\psi_2(x,\Delta t) = e^{-\frac{i(\hbar k)^2}{\hbar\ 2m}\Delta t}\psi_1(x,\Delta t)$

Diagonal in real space $\psi(x,\Delta t) = e^{-\frac{i}{\hbar}V(x)\frac{\Delta t}{2}}\psi_2(x,\Delta t)$

Why Excel?

Want a computing platform that all students can have access to.

Nuts and Bolts

$$f(\omega_i) = \int f(t)e^{-i\omega_i t}\,dt$$

Don’t calculate all components at once: too much typing.

Use iterative option of Excel.

Calculate the i^{th} component during the i^{th} iteration. Save for use later.

Data

Data × window

t_i

Window

cos ($\omega_n t_i$)

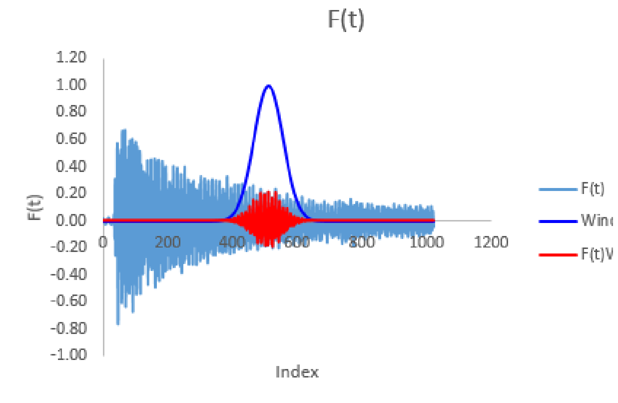
sin ($\omega_n t_i$)

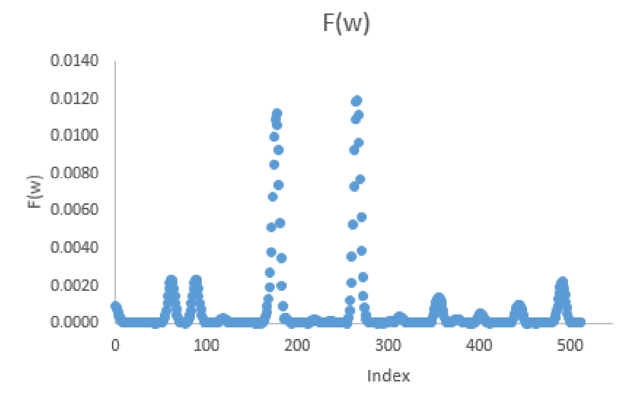
512	N=	1024

Re(f(w))	Im(f(w))	|f(w)|
8.94E-05	-2.21E-05	9.21E-05

index	f(t)	window W(t)	f(t)W(t)	cos(wt)	sin(wt)	f(t)W(t)cos(wt)	f(t)W(t)sin(wt)
0	0.017	1.60E-28	2.73E-30	1.00E+00	0.00E+00	2.73E-30	0.00E+00
1	0.014	2.06E-28	2.88E-30	-1.00E+00	6.14E-03	-2.88E-30	1.77E-32
2	0.004	2.64E-28	1.06E-30	1.00E+00	-1.23E-02	1.06E-30	-1.30E-32
3	0.004	3.39E-28	1.36E-30	-1.00E+00	1.84E-02	-1.35E-30	2.49E-32
4	-7.01E-04	4.34E-28	-3.04E-31	1.00E+00	-2.45E-02	-3.04E-31	7.47E-33
5	-0.013	5.56E-28	-7.23E-30	-1.00E+00	3.07E-02	7.23E-30	-2.22E-31
6	-0.02	7.12E-28	-1.42E-29	9.99E-01	-3.68E-02	-1.42E-29	5.24E-31
7	-0.026	9.12E-28	-2.37E-29	-9.99E-01	4.29E-02	2.37E-29	-1.02E-30
8	-0.027	1.17E-27	-3.15E-29	9.99E-01	-4.91E-02	-3.15E-29	1.55E-30
9	-0.019	1.49E-27	-2.83E-29	-9.98E-01	5.52E-02	2.83E-29	-1.56E-30
10	-0.023	1.91E-27	-4.39E-29	9.98E-01	-6.13E-02	-4.38E-29	2.69E-30
11	-0.017	2.44E-27	-4.14E-29	-9.98E-01	6.74E-02	4.13E-29	-2.79E-30
12	-0.005	3.11E-27	-1.56E-29	9.97E-01	-7.36E-02	-1.55E-29	1.14E-30
13	-0.003	3.97E-27	-1.19E-29	-9.97E-01	7.97E-02	1.19E-29	-9.49E-31
14	0.006	5.06E-27	3.04E-29	9.96E-01	-8.58E-02	3.03E-29	-2.61E-30
15	0.016	6.46E-27	1.03E-28	-9.96E-01	9.19E-02	-1.03E-28	9.49E-30
16	0.016	8.23E-27	1.32E-28	9.95E-01	-9.80E-02	1.31E-28	-1.29E-29
17	0.022	1.05E-26	2.31E-28	-9.95E-01	1.04E-01	-2.29E-28	2.40E-29
18	0.017	1.33E-26	2.27E-28	9.94E-01	-1.10E-01	2.25E-28	-2.50E-29
19	0.009	1.70E-26	1.53E-28	-9.93E-01	1.16E-01	-1.52E-28	1.78E-29
20	0.002	2.16E-26	4.32E-29	9.92E-01	-1.22E-01	4.29E-29	-5.29E-30
21	-0.002	2.74E-26	-5.49E-29	-9.92E-01	1.28E-01	5.44E-29	-7.05E-30
22	-0.017	3.49E-26	-5.93E-28	9.91E-01	-1.35E-01	-5.87E-28	7.98E-29
23	-0.023	4.43E-26	-1.02E-27	-9.90E-01	1.41E-01	1.01E-27	-1.43E-28

i	|f(w)|	Re(f(w))	Im(f(w))
0	8.84E-04	-8.84E-04	0.00E+00
1	8.51E-04	8.51E-04	-8.99E-07
2	7.58E-04	-7.58E-04	1.88E-06
3	6.26E-04	6.26E-04	-3.15E-06
4	4.79E-04	-4.79E-04	5.05E-06
5	3.41E-04	3.41E-04	-7.93E-06
6	2.26E-04	-2.26E-04	1.20E-05
7	1.42E-04	1.41E-04	-1.71E-05
8	8.65E-05	-8.35E-05	2.25E-05
9	5.61E-05	4.90E-05	-2.73E-05
10	4.29E-05	-3.01E-05	3.05E-05
11	3.79E-05	2.09E-05	-3.15E-05
12	3.51E-05	-1.74E-05	3.04E-05
13	3.28E-05	1.72E-05	-2.80E-05
14	3.17E-05	-1.91E-05	2.54E-05
15	3.26E-05	2.26E-05	-2.36E-05
16	3.54E-05	-2.70E-05	2.28E-05
17	3.89E-05	3.18E-05	-2.24E-05
18	4.14E-05	-3.58E-05	2.08E-05
19	4.15E-05	3.79E-05	-1.67E-05
20	3.88E-05	-3.76E-05	9.39E-06
21	3.47E-05	3.47E-05	6.10E-07
22	3.22E-05	-3.00E-05	-1.16E-05
23	3.29E-05	2.48E-05	2.15E-05





Be Careful

Excel calculates from left to right and top down for each iteration.

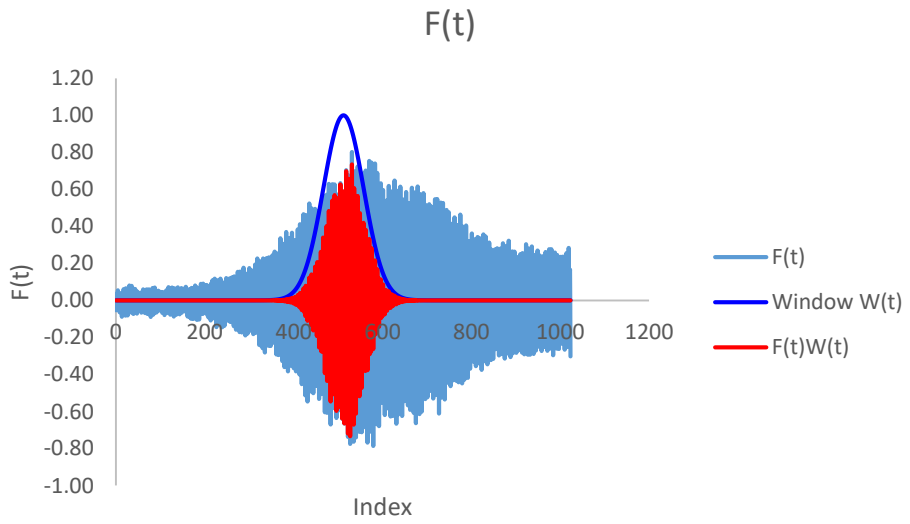
So make sure instructions are in the correct order.

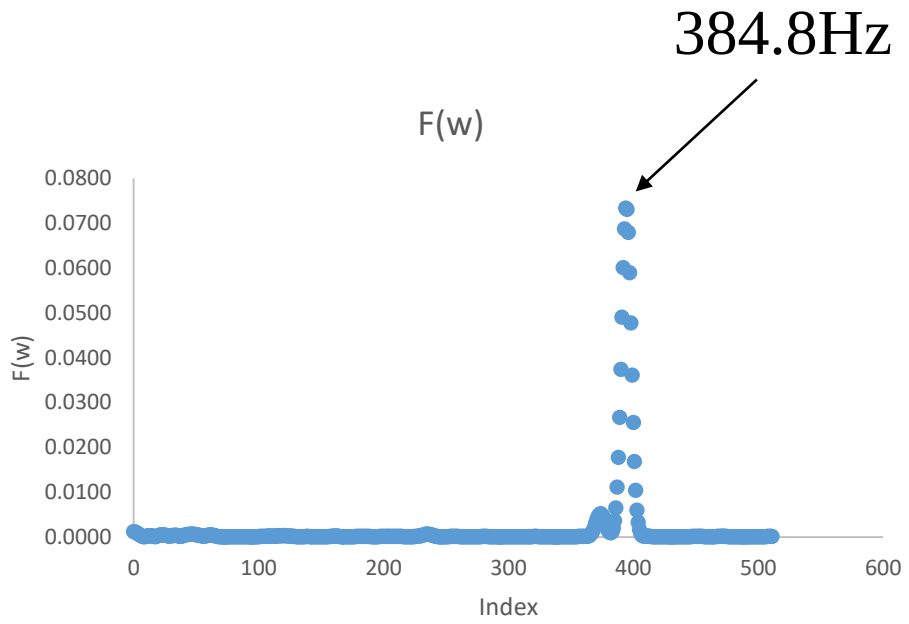
Not FFT

Can program to do FFT.

But too complex, and the gain in computation time for the data size is not worth it.

Tuning Fork (384 Hz)

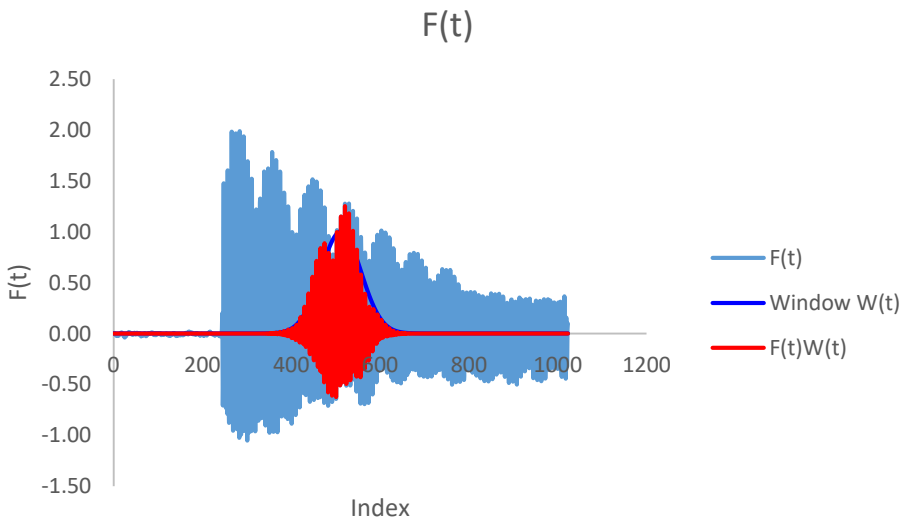


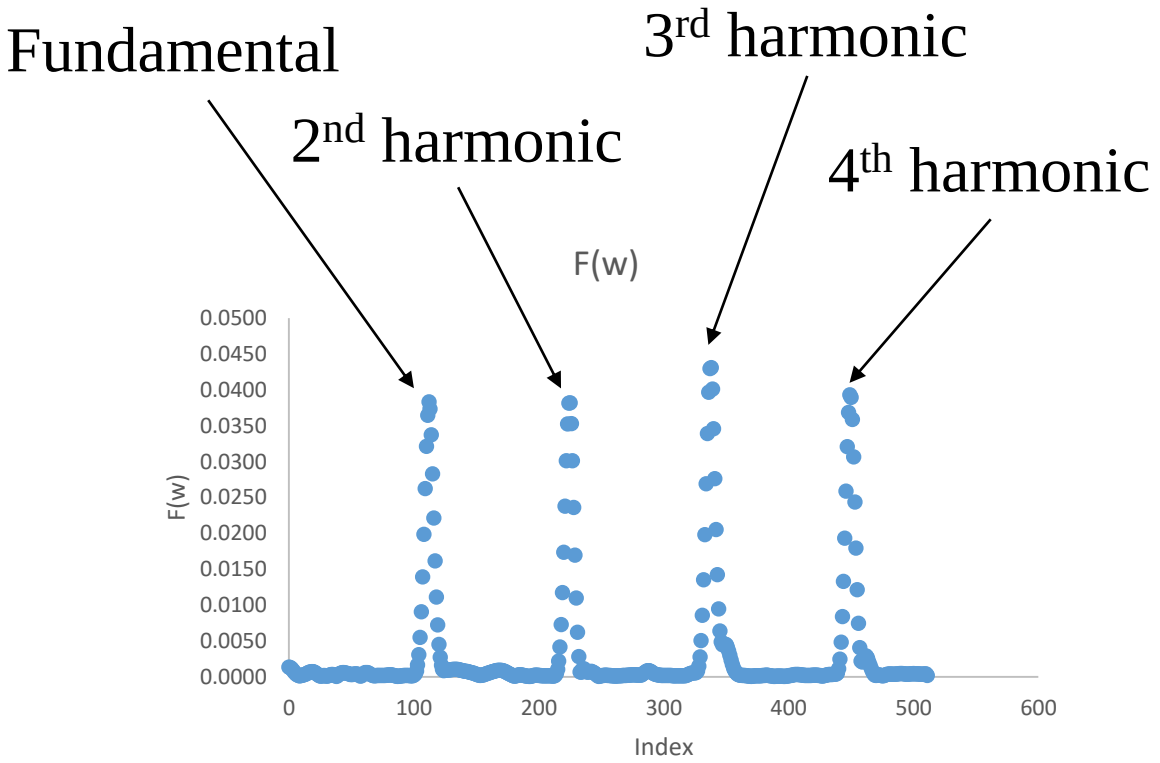


Usual Standing Wave Experiment

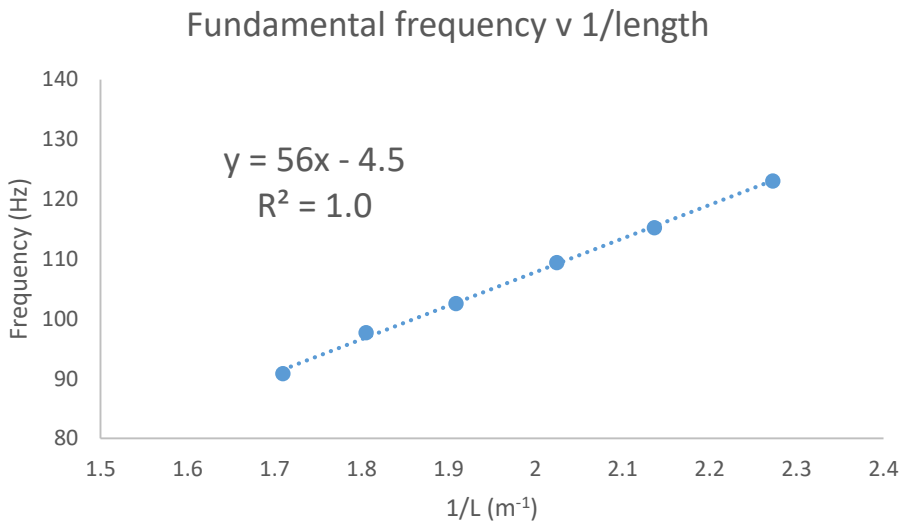
Wouldn’t it be more interesting to do an experiment with a musical instrument?

Standing Wave on a Guitar

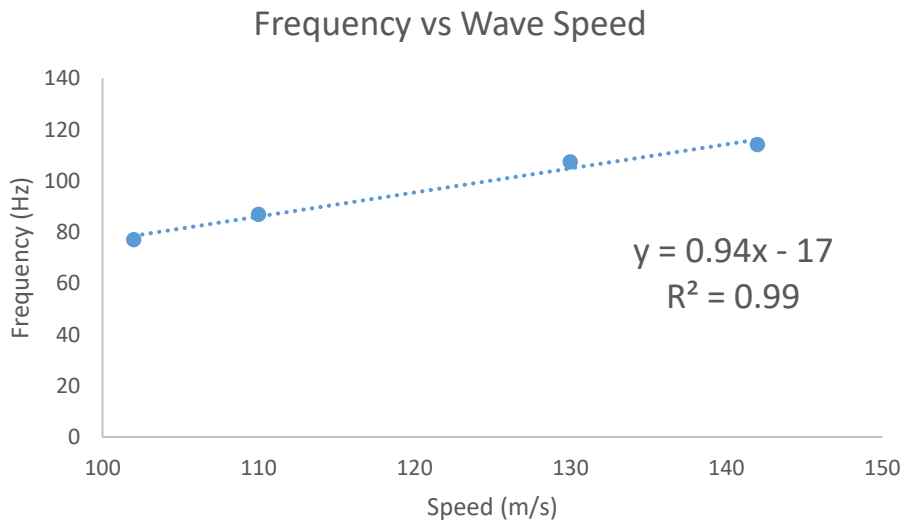




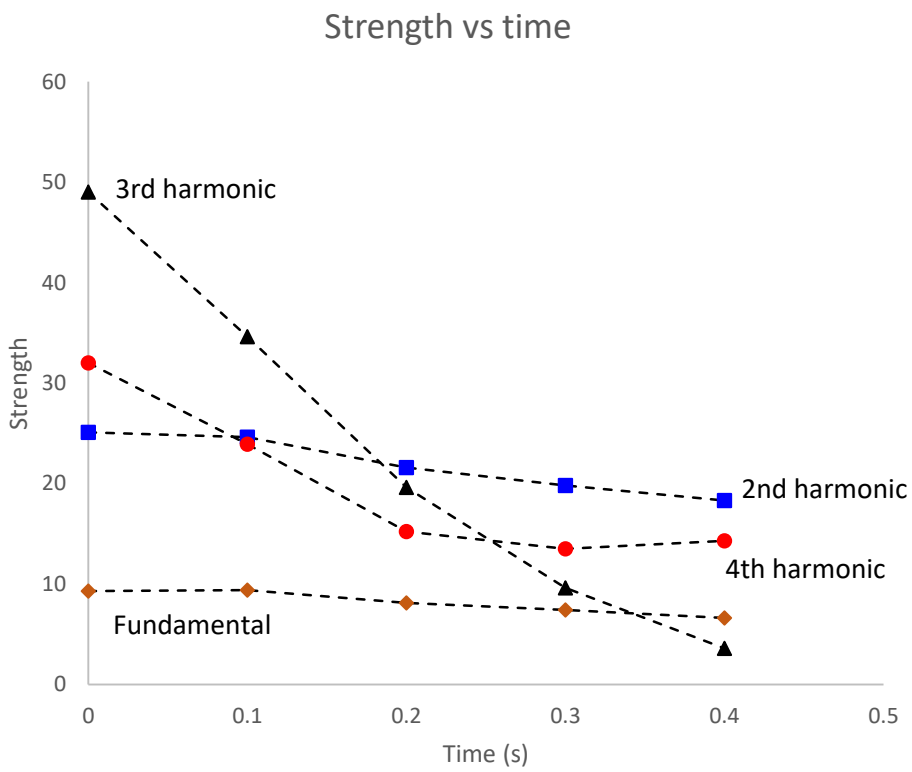
Frequency vs 1/Length



Frequency vs Wave Speed



Strength vs Time



What Else?

Pluck the string at different positions to see how that affects the composition of the different harmonics.

Benefits Over Old Experiment

Students can see how physics works in a real life application.

Teach the principle of superposition of waves.

Difficulty

We only have one guitar.

Solution: ask students to bring in their own if they have one.

Thank you