Riding on a Light Beam

The Brehme Angle and The Velocity Triangle

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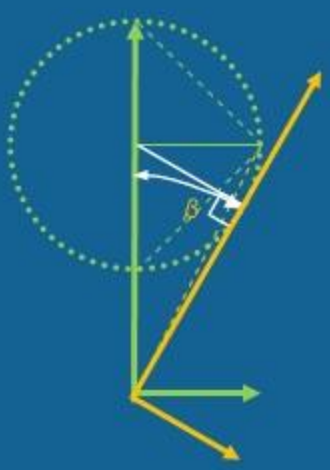

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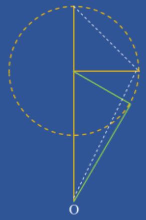
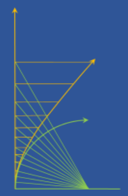
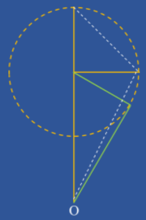


AGENDA

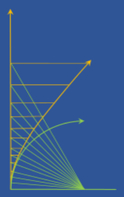


- The Lorentz Transformation
- Other Graphical Solutions
  - Minkowski Diagram
  - Brehme Diagram

- The Brehme Angle & Velocity Triangle
- Rotated Reference Frames, Translating at *c* in Direction of Time
- Events, Observations and Measurements
- Communications Between Reference Frames by Doppler Shift
- Multiple Reference Frames and the Invariant $c\tau$



The Lorentz Transformation



- SIMPLE ENOUGH! TRANSFORMS MEASUREMENT FROM ONE REFERENCE FRAME TO ANOTHER

$$x_2 = \frac{x_1 + \left(\frac{v}{c}\right) ct_1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$ct_2 = \frac{ct_1 + \left(\frac{v}{c}\right) x_1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

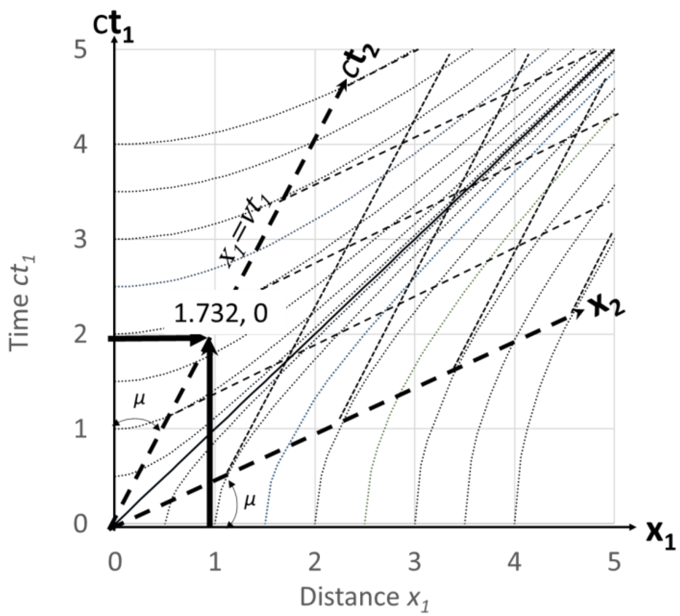
- FOR $v << c$, $x << ct$, BECOMES THE GALILEAN TRANSFORM:

$$x_2 \cong x_1 + vt_1$$
$$ct_2 \cong ct_1$$

- BUT WHAT DOES IT MEAN?

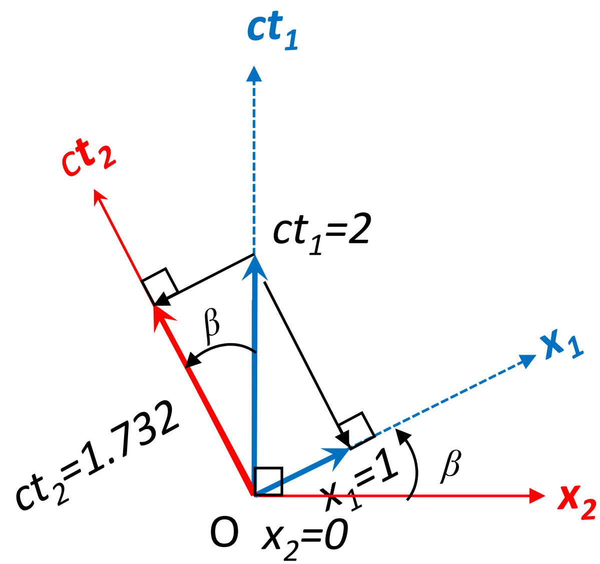
GRAPHICAL SOLUTIONS

Minkowski and Brehme Diagrams



Minkowski Diagram

World Line *x*=*vt* and Constant $c\tau = \sqrt{(ct_1)^2 - x_1^2}$



Brehme Diagram

Reference Frame Orthogonalities

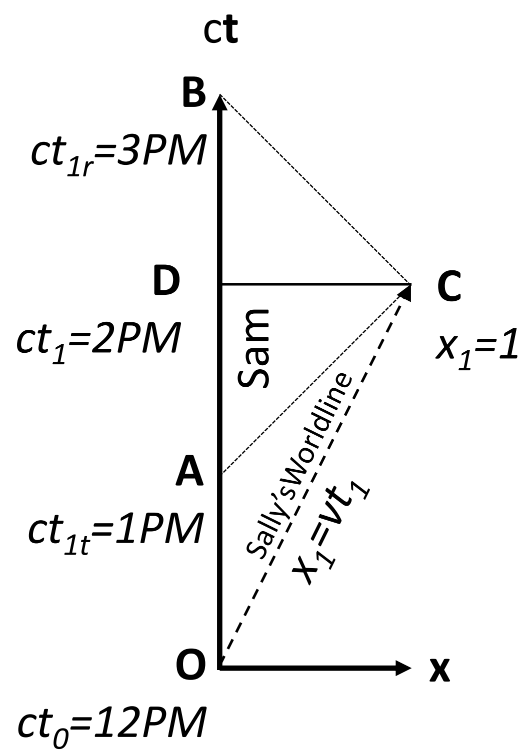
Expanded/Reduced by Brehme Angle B=$\sin^{-1}(v/c)$

WHAT TIME IS IT, SALLY?

- SALLY LEAVES EARTH
  - ct_0 =12PM
  - $v/c=0.5c$

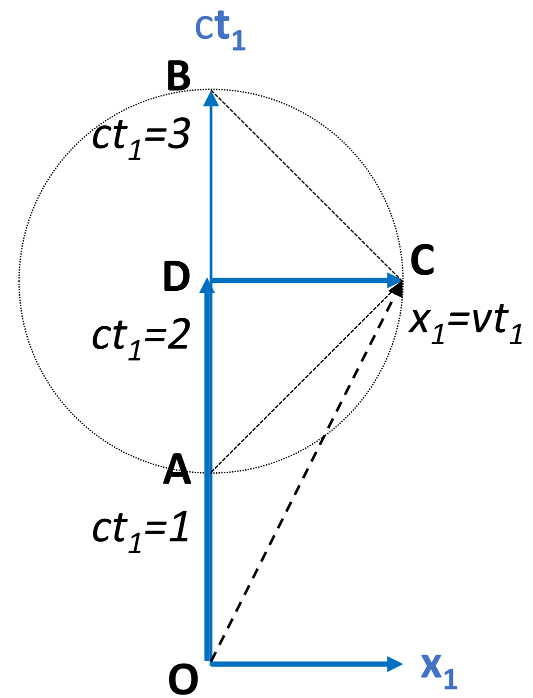
- SAM TRACKS SALLY BY RADAR ON HER WORLDLINE $x_1=vt_1$
- AT ct_t = $1PM$, SAM ASKS SALLY WHAT TIME IT IS
- AT ct_r $=3PM$ SAM GETS HER REPLY AND COMPUTES BY RADAR EQUATION:
  - $x_1= (ct_{1r}-ct_{1t})/2$ $=1$ *light hour*
  - $ct= (ct_{1r}+ct_{1t})/2$ $=2PM$
  - $v=x/ct$ $=0.5$

- *BUT SALLY SAID HER CLOCK READ 1:43PM WHEN SHE GOT HIS REQUEST!*



THE BREHME ANGLE AND THE VELOCITY TRIANGLE

- DRAW A CIRCLE ABOUT D=ct_1, OF RADIUS x_1
  - A is Time of Emission of Radar Pulse, Sam's Query
  - B is Time of Receipt of Radar Pulse , and Sally's Reply
  - C is Sally's Location Determined by A and B
    - $x_1=(3-1)/2=1$
    - $ct_1=(3+1)/2=2$
    - $v=x_1/ct_1=0.5c$



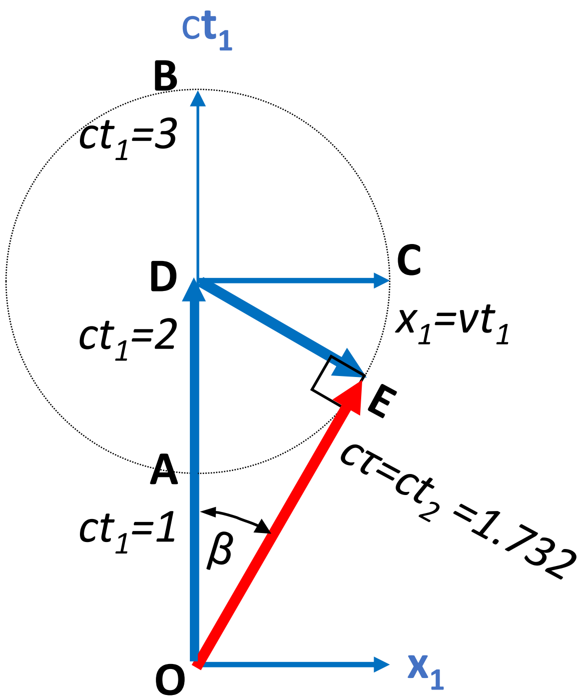
THE BREHME ANGLE AND THE VELOCITY TRIANGLE

- DRAW LINE FROM O TANGENT TO CIRCLE AT E
- OE IS THE INVARIANT

$$c\tau = ct_2 = \sqrt{(ct_1)^2 - x_1^2} = ct_1\sqrt{1 - \left(\frac{v}{c}\right)^2}$$

- VELOCITY TRIANGLE IS ODE
- OE SEPARATED FROM SAM'S TIME AXIS OD BY *Brehme Angle $\beta = \sin^{-1}(v/c)$*

- E, SALLY'S ACTUAL TIME OF RECEIPT AND TRANSMISSION, IS ALSO ON THE INFERENTIAL CIRCLE

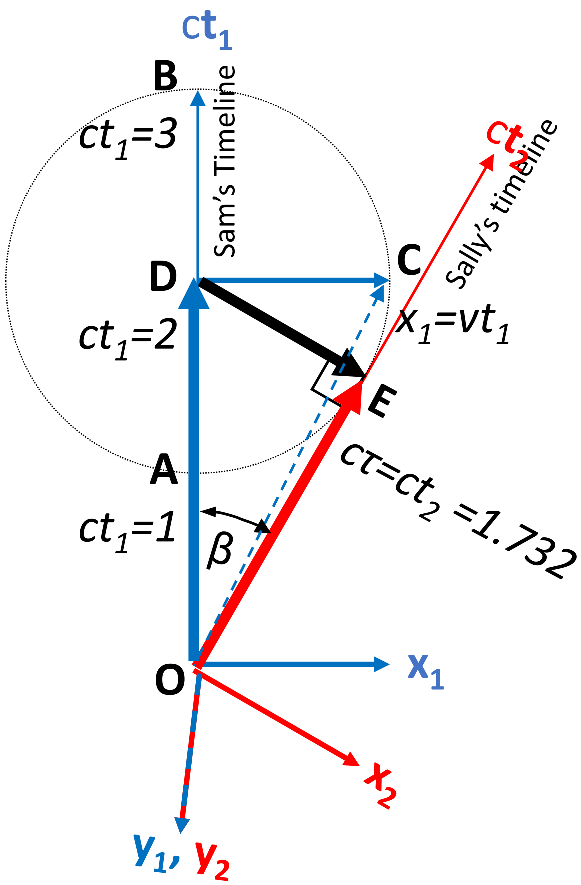


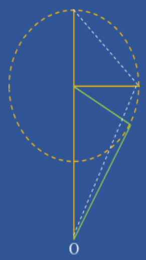
THE BREHME ANGLE AND THE VELOCITY TRIANGLE

- VELOCITY TRIANGLE ODE:
  - Sally's ct_2 Time Axis through OE
  - Sally's x_2 Spatial Axis Normal to OE, in Direction of Motion
- WORLDLINES AND TIMELINES
  - Minkowski Worldline OC
    - Locus of Successive Measurement of Sally by Sam
  - Timelines OD and OE (NEW)
    - Locus of Proper Clock Ticks from O by Sam and Sally
- TRIGONOMETRIC LORENTZ TRANSFORM

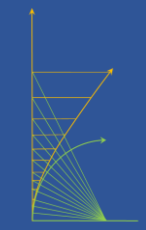
$$x_1 = \frac{x_2 + ct_2 \cdot \sin(\beta)}{\cos(\beta)}$$

$$ct_1 = \frac{ct_2 + x_2 \cdot \sin(\beta)}{\cos(\beta)}$$

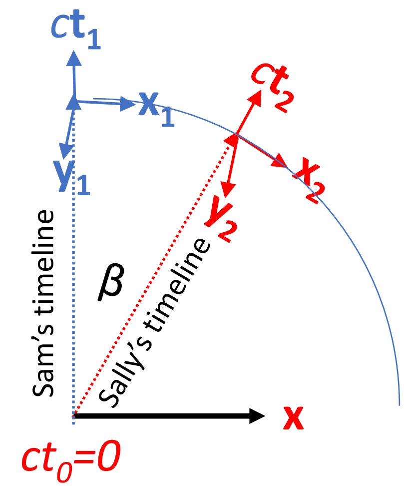


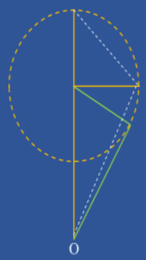


ROTATED REFERENCE FRAMES PROPAGATING AT *c*.

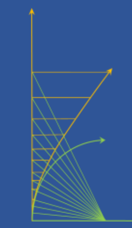


- LIGHT PROPAGATES AS A SPHERE EXPANDING AT *c* IN FOUR-DIMENSIONAL SPACE
  - $(c\Delta t)^2 = \Delta x^2 + \Delta y^2 + \Delta z^2$
- ALL MATTER PROPAGATES AT PROPER VELOCITY *c* IN A SINGLE DIRECTION
  - That direction defines its timeline unit vector *c***t**
  - Spatial axes' unit vectors **x, y, z** normal to c**t**.
- RELATIVE VELOCITY IS PROJECTION OF ONE BODY'S PROPER VELOCITY ONTO SPATIAL PLANES OF ANOTHER
  - Through *β* and Lorentz Transform
  - Does Not Exist as an Independent Variable
  - Maximum Relative Velocity *c* at β=90º





EVENTS, OBSERVATIONS & MEASUREMENTS



DEFINITION

- NORMALLY NEGLIGIBLY SEPARATED IN TIME OR SPACE FOR $v << c$: NOT TRUE FOR HIGHER v*!*
- <u>EVENT</u>: Proper Event with Zero Spatial Coordinates that Generates Light (Time of Transmission)
- <u>OBSERVATION</u>: Proper Time (Zero Spatial Coordinates) of Receipt of Light from an Event in One Reference Frame By an Observer in Another Reference Frame (Time of Receipt)
- <u>MEASUREMENT</u>: Determination of ct, x Coordinates in Observer's Reference Frame.
  - <u>AFFECTED BY THE DOPPLER SHIFT DUE TO RELATIVE MOTION BETWEEN EVENT AND OBSERVER!</u>

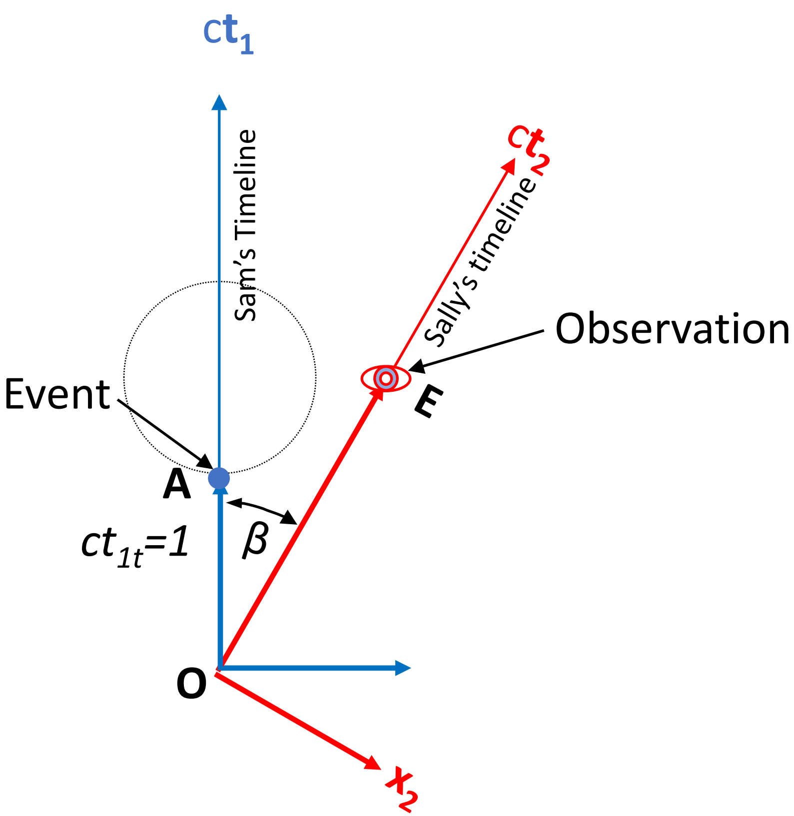
COMMUNICATING BETWEEN REFERENCE FRAMES

Sam's Request to Sally

- Sam's Request is Doppler Shifted Along Path AFE from the Time of His Transmission to Sally's Time of Receipt:

$$ct_{2r} = ct_{1t}\frac{1+\sin(\beta)}{\cos(\beta)}$$

Equal units of space in the reference frame of origination, to equal units of time in reference frame of receipt.



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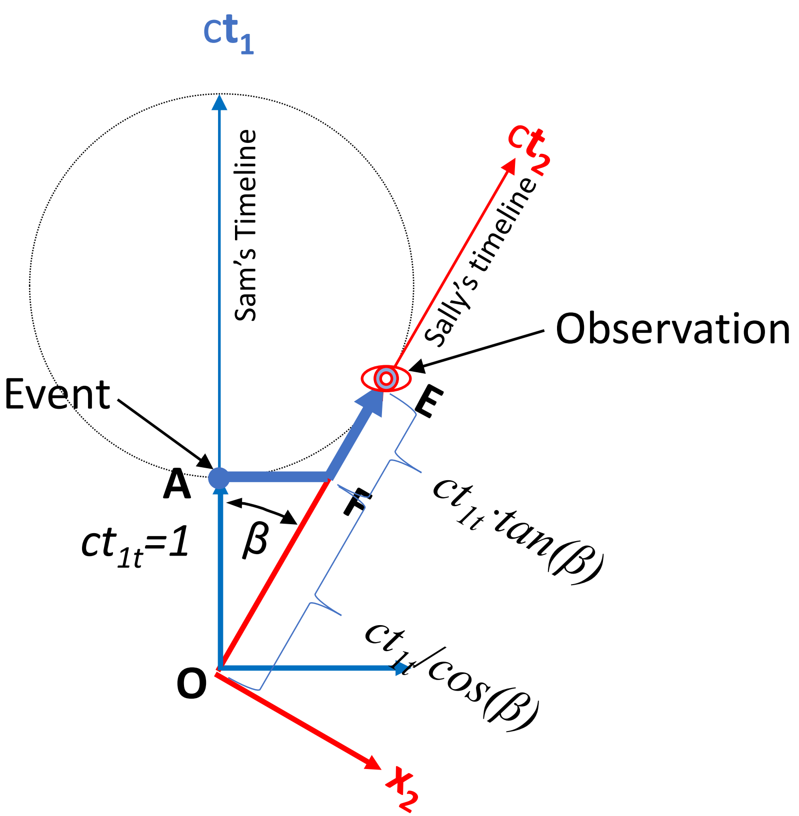
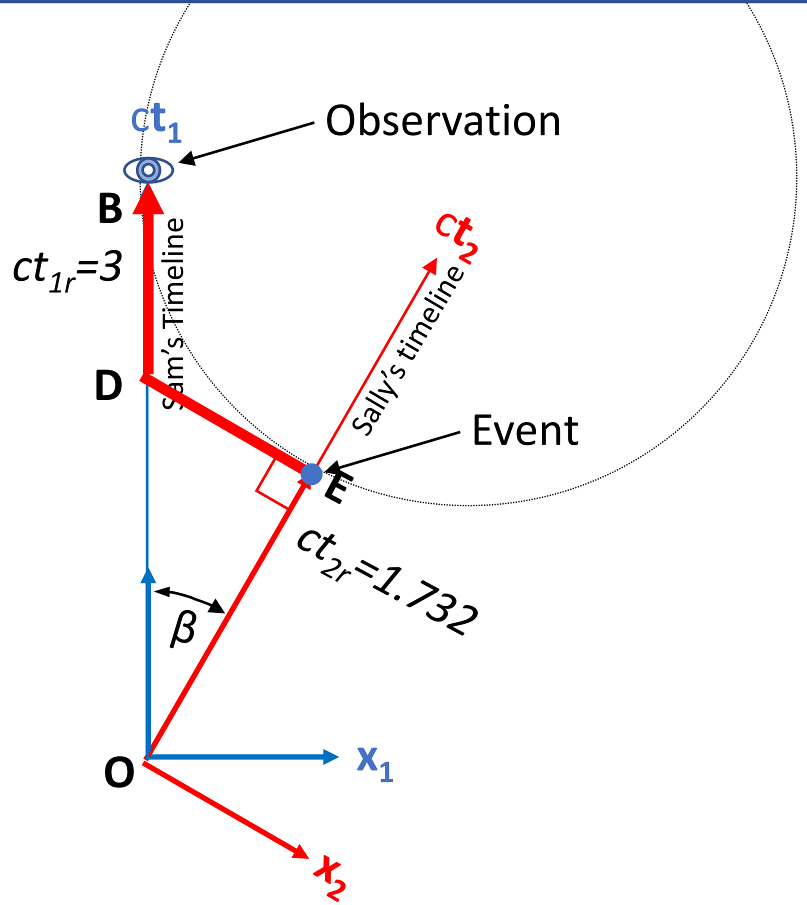


COMMUNICATING BETWEEN REFERENCE FRAMES

Sally's Reply to Sam

- Sally's Reply is Doppler Shifted Along Path EDB from the Event of Her Transmission to Sam's Time of Observation:

$$ct_{1r} = ct_{2t}\frac{1+\sin(\beta)}{\cos(\beta)} = 3$$



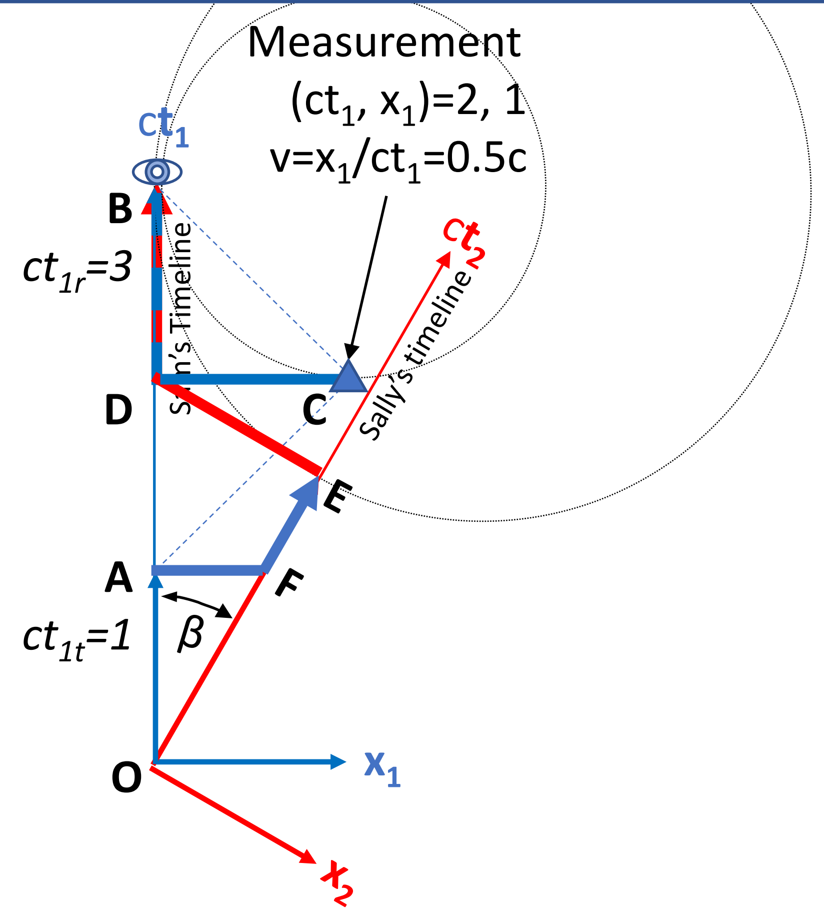
COMMUNICATING BETWEEN REFERENCE FRAMES

Sam Determines Sally's Location

- Sam Measures Sally's Position From the Event of His First Transmission and His Observation of Sally's Reply on His Timeline :

$$x_1 = \frac{ct_{1r} - ct_{1t}}{2} = 1$$
$$ct_1 = \frac{ct_{1r} + ct_{1t}}{2} = 2$$
$$v = \frac{x_1}{ct_1} = 0.5$$

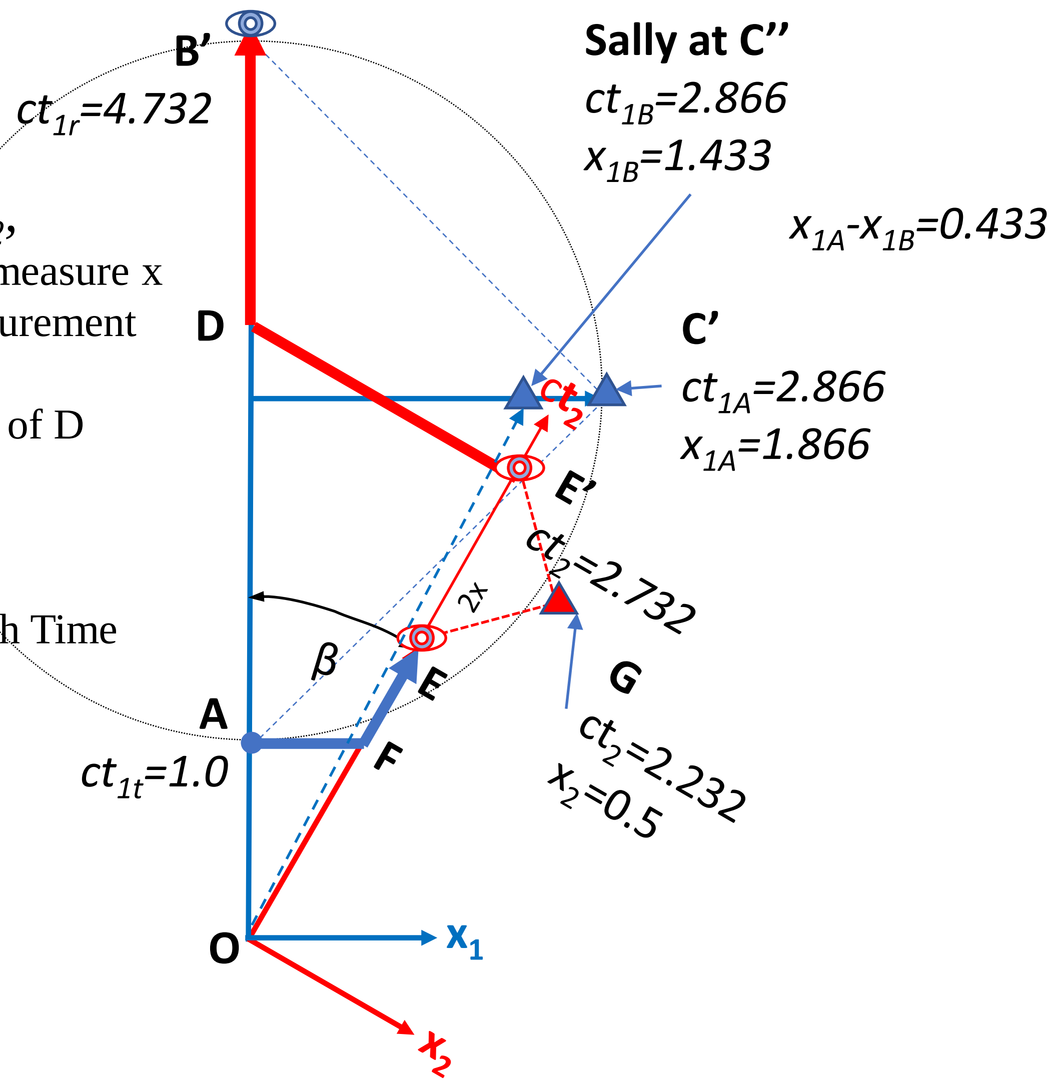
- Light emitted by Sally at E arrives at B simultaneous with light from a fixed point at C: Sally is "at" *C*.



TRANSFORMATION OF MEASUREMENTS

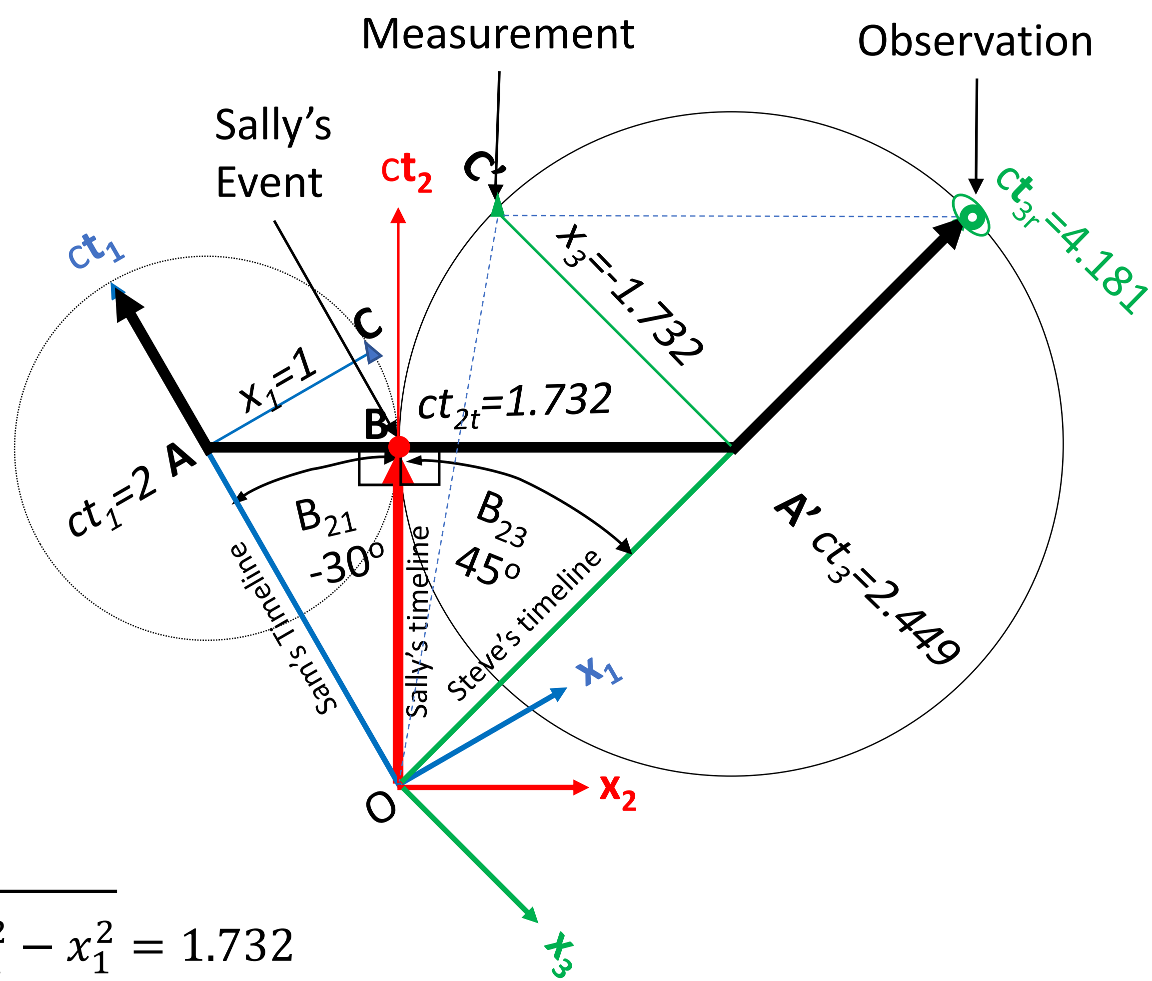
- FOR ARBITRARY ct_2, x_2,
  - Sam must request Sally to measure x
  - Sally Must Complete Measurement and Communicate to Sam
  - H is the Lorentz Transform of D

- DISTANCE MAPPED ONTO ORIGINATOR'S TIMELINE
  - Doppler-Shifted, Along with Time

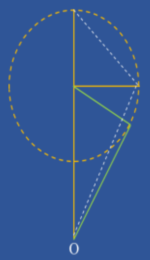


COMMUNICATING BETWEEN REFERENCE FRAMES

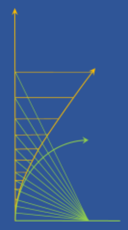
All Reference Frames Agree on Proper Time of the Event



$$c\tau = \sqrt{ct_3^2 - x_3^2} = \sqrt{ct_1^2 - x_1^2} = 1.732$$



SUMMARY



- VELOCITY TRIANGLE
  - Separates Event, Observation and Measurement
  - Eliminates “Rubber Rulers” and “Asynchronous Clocks”
  - The Measurement of One Clock by Another Affected by Relativistic Doppler Shift
- PROVIDES MORE CLEAR, LESS MYSTERIOUS DEMONSTRATION OF ALL ASPECTS OF RELATIVITY USING SIMPLE GEOMETRY
  - Additive Velocities
  - Mass, Momentum and Energy
- CAN BE APPLIED TO ACCELERATED (NON-INERTIAL) SYSTEMS