Projectile Motion Revisited

Analysis of horizontal projectiles using a simple apparatus

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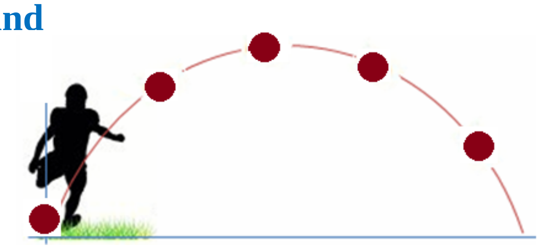
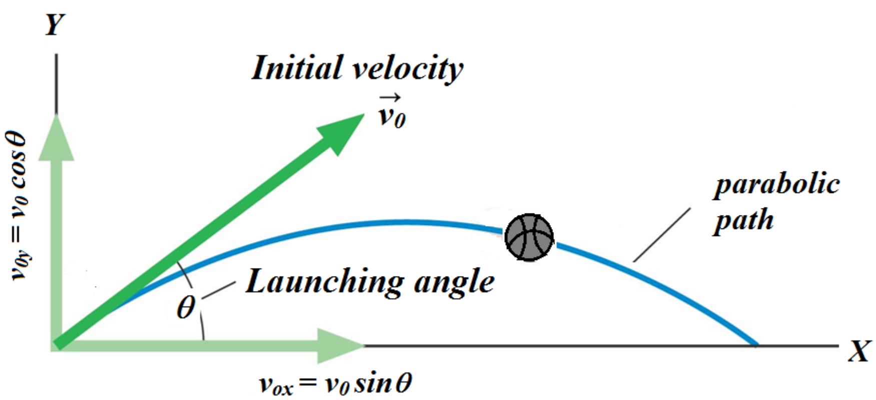


Abstract:

Many students who are taking introductory physics course find it difficult to grasp concepts in projectile motion as it is discussed early in the course. A different approach is used to introduce projectile motion experiment after the energy concepts have been taught and at a later stage of the course. A simple and easy to make apparatus is introduced to discuss horizontal projectile motion. A hula hoop is cut and fixed vertically to a wooden base which rests on a table. A small ball drops from the upper end of the hoop emerges from the bottom end as a horizontal projectile and lands on a carbon paper. The velocity can be determined from the height of the table, the horizontal distance traveled and using kinematic equations. The theorem of conservation of energy can also be used to predict the initial velocity of the projectile. The ways of extending the apparatus to discuss additional concepts involving horizontal projectile motion will also be presented.

Projectile Motion

A **projectile** is an object moving in two dimensions under the influence of Earth's **gravity**.

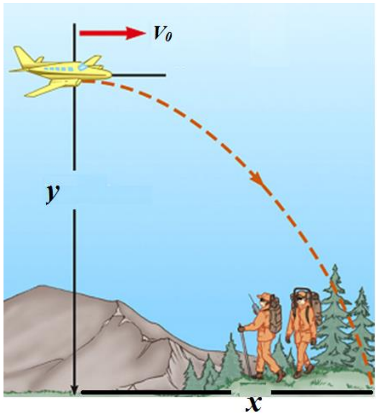


Kinematic Equations for Projectile Motion

$$x = v_0 t + \frac{1}{2} a t^2 \qquad v = v_0 + at \qquad v^2 = v_0^2 + 2ax$$

$a_x = 0$ *and* $a_y = -g = -9.8\ m/s^2$	
horizontal motion	*vertical motion*
$a_x = 0$ and v_x = constant	$a_y = -g$ = constant
$v_x = v_{0x}$	$v_y = v_{0y} - gt$
$x = v_{0x} t$	$y = v_{y0} t - \frac{1}{2} g t^2$
	$v_y^2 = v_{y0}^2 - 2gy$

Horizontal projectile



Horizontal Velocity, $v_x = v_0$

Horizontal distance, $x = v_0 t$

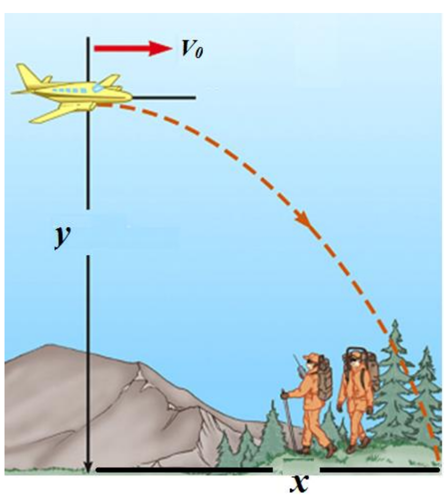
Vertical Velocity, $v_y = v_0 - gt$

Vertical distance, $y = v_{y0} - \frac{1}{2}gt^2$

$-y = -\frac{1}{2}gt^2$ $\quad$ $y = \frac{1}{2}gt^2$

Photo credit: Serway, R. A., & Vuille, C. (2008). College physics: Volume 1.

Finding the initial horizontal velocity of a projectile



$$x = v_0 t \rightarrow v_0 = \frac{x}{t}$$

$$y = \frac{1}{2}gt^2 \rightarrow t = \sqrt{\frac{2y}{g}}$$

$$v_0 = x\sqrt{\frac{g}{2y}}$$

Photo credit: Serway, R. A., & Vuille, C. (2008). College physics: Volume 1.

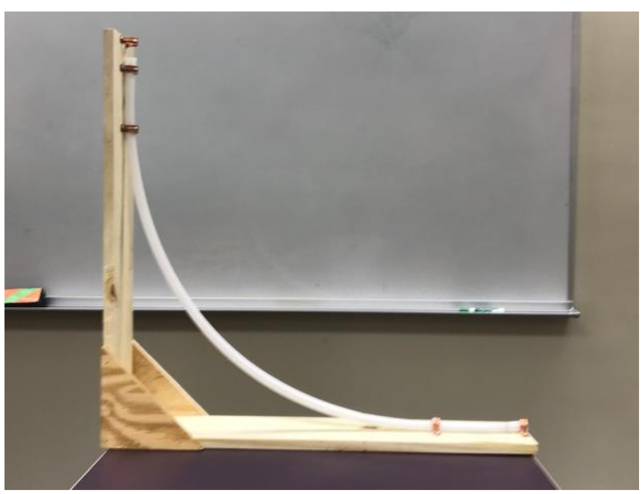
LAB EXERCISE:

Finding the initial horizontal velocity of a projectile

Three methods:
a) Using kinematic equations
b) Using conservation of energy
c) Drawing a graph

***Smart timers (electronic timers) or photogates are not required as time of flight is not measured*.**

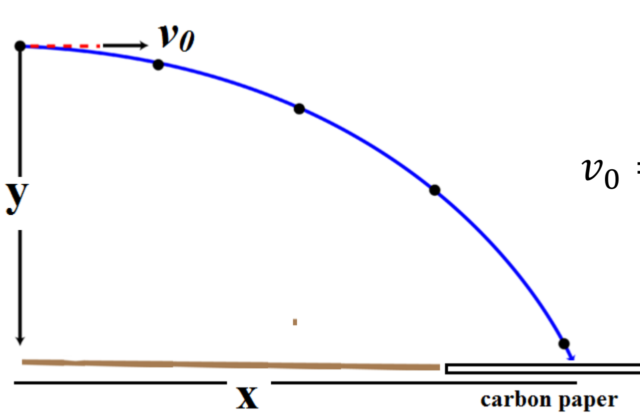
Apparatus:



A partially cut hula hoop is fixed vertically to a wooden base which rests on a table. A small ball drops from the upper end of the hoop emerges from the bottom end as a horizontal projectile and lands on a carbon paper.

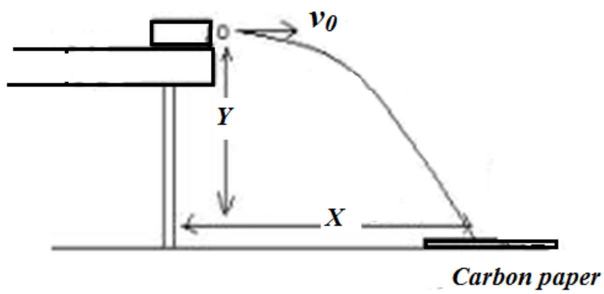
In this lab, you will drop a steel ball through a pipe. The ball emerges from the end of the pipe with an initial horizontal velocity and land on a carbon of paper.

The pipe is horizontal at the end of its track, so that steel ball exits the pipe with a horizontal velocity.

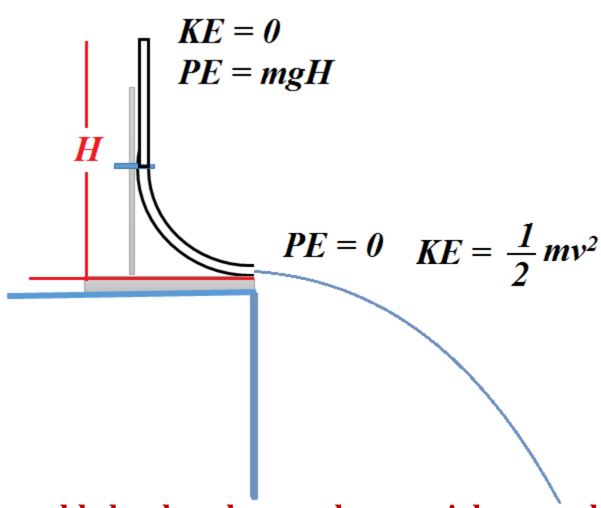


$$v_0 = x\sqrt{\frac{g}{2y}}$$

Place a sheet of carbon paper on a top of a white paper (inky-side down) on the floor.
Do a few trials to locate the point where the ball hits the floor so that carbon paper can be placed at the correct location to get a mark from the falling ball.



Predicting the initial velocity using conservation of energy.



Assume table level as the zeroth potential energy level. Measure the height H of the top of the track relative to the table.

If there were no friction between the steel ball and the track, the mechanical energy is conserved.

Apply the theorem of Conservation of mechanical Energy

$(KE + PE)_{initial} = (KE + PE)_{final}$

$$K_i + U_i = K_f + U_f$$
$$0 + mgh = \tfrac{1}{2}mv^2 + 0$$
$$v = \sqrt{2gh}$$

Analysis:

Initial velocity of the ball from kinematic equations:

---------------- m/s^2

Initial velocity of the ball from the energy conservation:

-------------------- m/s^2

Find the percent discrepancy of the two measurements

Analysis:

Find the initial energy at the top of the track

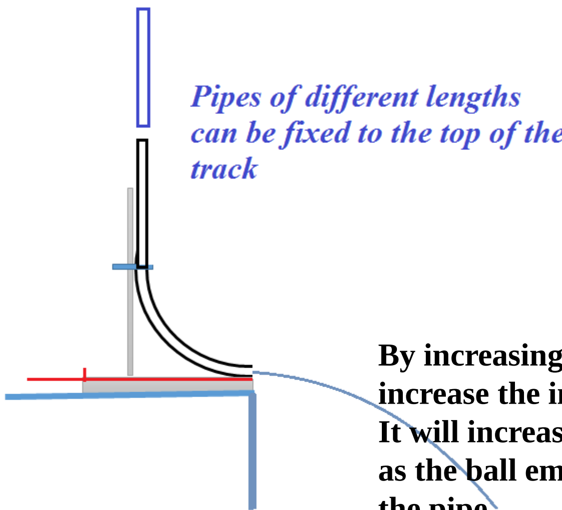
Using the velocity of the projectile calculated from the height of the table and the horizontal distance, find the final energy of the ball as it emerges from the bottom of the track.

Are they equal?

If not, find the energy loss due to friction

If the length of the pipe is 1.0 m, what is the average frictional force

Increasing the initial velocity:



By increasing the height, we can increase the initial potential energy. It will increase the initial velocity as the ball emerges from the end of the pipe

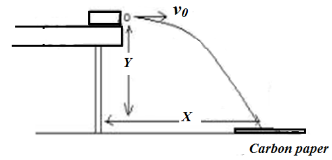
STOP TO THINK!

What would be the effect on maximum range if the initial velocity is doubled?

Finding the initial velocity by graphical method:

$x = v_0 t$

$y = \frac{1}{2} g t^2 \rightarrow h = \frac{1}{2} g t^2 \rightarrow t = \sqrt{\frac{2h}{g}}$



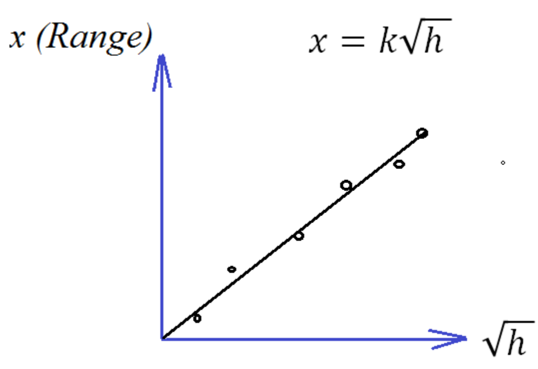
$x = v_0 \sqrt{\frac{2h}{g}} \rightarrow x = \frac{v_0\sqrt{2}}{\sqrt{g}}\sqrt{h} \rightarrow x = k\sqrt{h}$

where k is constant $k = \frac{v_0\sqrt{2}}{\sqrt{g}}$

If we plot a graph x versus $\sqrt{h}$ we can get a straight line.

The slope of the graph = $k = \frac{v_0\sqrt{2}}{\sqrt{g}}$

From slope we can find v_0 (initial velocity)



The slope of the graph = $k = \frac{v_0\sqrt{2}}{\sqrt{g}}$

Increasing h (the height of the table):

Place the apparatus at different levels