



Implementing New Web Capabilities to Develop Updated Laboratory Procedures Brianna Wachter (briannatwachter@gmail.com) and Perry Wood (pwood@frederick.edu) Department of Science, Frederick Community College

KEPLER'S 3RD LAW OF PLANETARY MOTION



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Goal: Create a revised implementation of the Moons of Jupiter lab using new web capabilities, specifically, Horizons Web-Interface.





Johannes Kepler

- German astronomer and mathematician
- 1571-1630
- Believed there was an inherent order to the universe and that "those laws [of nature] are within the grasp of the human mind..."
- Work Included: Rudolphine Tables, The Kepler Conjecture, and Kepler's Laws of Planetary Motion

Kepler's Laws of Planetary Motion



The square of the orbital period of a planet is directly proportional to the cube of the semi-major axis of its orbit

$$\left(\frac{T_1}{T_2}\right)^2 = \left(\frac{a_1}{a_2}\right)^3$$

T = perioda = semi - major axis



The Significance of Kepler's Laws



Supported the model of the Sun as the center of the Solar System



Inspired Isaac Newton to explain motion with the Law of Universal Gravitation







Contemporary Laboratory Exercises in Astronomy

- Designed by Gettysburg College in the 1990s
- A resource of free software-based laboratory exercises for astronomy students
- Still available for download but no longer being updated

CLEA Exercise - The Revolution of The M December 1, 2018 0 Hrs, 00 Min, 00.0 Sec Jul. Day: 2458453.500000 Data Interval: 24.00000 Hrs

Collect the Data

In the CLEA software, students record the position of the four moons in relation to Jupiter

-Graph the position versus time

-Fit this data to a sine function

-Record period and amplitude

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r Jupiter		Act Sin A
		1±1.0
•		$\frac{COS'\Theta = 1}{1 + 1 = sec^{2}\Theta}$ $\frac{\Theta}{\Theta} = coses^{2}\Theta$ $\frac{2sin A cos A}{2sin^{2}A - sin^{2}A}$ $\frac{cos^{2}A - sin^{2}A}{1 - 2sin^{2}A}$
300X Cont. 400X Next	III. Ganymede X: 450, Y: 193 X = 1.69W (Jup. Diam.) <u>V R</u> ecord	Elan A Can A Sin A con B ± cos A sin B MA con B ± sin A sin B Tan A stand Tan A stand

Analyze the Data

Calculate the Mass of Jupiter

Use the following equation to calculate the mass of Jupiter: $M = \frac{a^3}{p^2}$



Collect the Data

In the CLEA software, students record the position of the four moons in relation to Jupiter Analy raph the

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Analyze the Data

-Graph the position versus time

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Collect the Data

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-Graph the position versus time

-Fit this data to a sine function

-Record period and amplitude

Analyze the Data

Calculate the Mass of Jupiter

Use the following equation to calculate the mass of Jupiter: $\frac{a^3}{p^2}$ M =

An Updated Approach...



Collecting Data

- Horizon's Web-Interface by NASA's Jet **Propulsion Laboratory**
- Set location and observation intervals
- Collect Julian date, light minutes, right ascension, and declination

Declination and Right Ascension

Celestial Sphere

- No set axis
- Centered about Earth
- Relates the position of objects by measuring their passage

Right Ascension

- "Longitude"
- Measured in hours with 24 hours being 1 rotation

Declination

- "Latitude"
- Celestial equator: declination of 0°
- North celestial pole: declination of +90°
- South celestial pole: declination of -90°


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Revised: Jun 17, 2016
                        Europa / (Jupiter)
                                                        502
SATELLITE PHYSICAL PROPERTIES:
                  = 1565 +- 8 Density (g cm^-3) = 2.99 +- 0.05
 Radius (km)
 Mass (10^20 kg )
                  = 479.7 +- 1.5 Geometric Albedo
                                              = 0.6
SATELLITE ORBITAL DATA:
 Semi-major axis, a (km) = 671.079 (10^3) Orbital period
                                              = 3.551810 d
                                Rotational period = Synchronous
 Eccentricity, e
                = 0.0101
 Inclination, i (deg) = 0.464
Ephemeris / WWW_USER Tue Nov 27 02:51:45 2018 Pasadena, USA
                                              / Horizons
Target body name: Europa (502)
                                     {source: jup310}
Center body name: Earth (399)
                                    {source: DE431mx}
Center-site name: GEOCENTRIC
: A.D. 2018-Nov-01 00:00:00.0000 UT
Start time
          : A.D. 2018-Nov-08 00:00:00.0000 UT
Stop time
Step-size
          : 360 minutes
Target pole/equ : IAU_EUROPA
                                     {West-longitude positive}
Target radii : 1562.6 x 1560.3 x 1559.5 km
                                   {Equator, meridian, pole}
Center geodetic : 0.00000000,0.00000000,0.00000000 {E-lon(deg),Lat(deg),Alt(km)}
Center cylindric: 0.00000000,0.00000000,0.00000000 {E-lon(deg),Dxy(km),Dz(km)}
Center pole/equ : High-precision EOP model
                                    {East-longitude positive}
Center radii : 6378.1 x 6378.1 x 6356.8 km
                                    {Equator, meridian, pole}
Target primary : Jupiter
Vis. interferer : MOON (R_eq= 1737.400) km
                                    {source: DE431mx}
Rel. light bend : Sun, EARTH
                                    {source: DE431mx}
Rel. lght bnd GM: 1.3271E+11, 3.9860E+05 km^3/s^2
Atmos refraction: NO (AIRLESS)
RA format
           : DEG
          : JD
Time format
EOP file
           : eop.181126.p190217
EOP coverage : DATA-BASED 1962-JAN-20 TO 2018-NOV-26. PREDICTS-> 2019-FEB-16
Units conversion: 1 au= 149597870.700 km, c= 299792.458 km/s, 1 day= 86400.0 s
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Collecting Data

- Horizon's Web-Interface by NASA's Jet **Propulsion Laboratory**
- Set location and observation intervals
- Collect Julian date, light minutes, right ascension, and declination
- Copy from plain text output

4	A	В	C	D	E	F _	G	н	I 1	J	К	L	M	N	0			P			Q		F
Je	ılian Date	Jupiter RA	Jupiter DEC	Light Min		lo RA	lo Dec		Time (days	<u>Δθ</u> -	Fit'	Diff ^a											
L	2458527.5	258.58783	-22.39922	47.331951	2	258.61246	-22.40167							w (rad/se	20	0 <				> /	Average Dis	stance	(m)
Ŀ	2458527.63	258.60847	-22.40071	47.316641		258.6385	-22.40294																
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	2458527.88	258.64922	-22.40381	47.285695	2	58.67262	-22.40444									0.5							
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_	2458528.25	258.70327	-22,40612	47.23370	2	200.0007	-22.40537)	χ						- P	'eriod (sec	ondsj	
-	2408028.38	250.72343	-22.40340	47.224013	Z	250. r0343	-22.40102							Re	set	H							
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	2406023.13	258 86971	-22.41010	47.131001	2	58 89309	-22.42024							0.7									
	2400020.20	258,88966	-22.41040	47 101311	2	258 91936	-22.42304																
	2459529.5	258 90976	-22 42213	47.085953	2	58 93999	-22 42378							0.8									
	2458529.63	258 92988	-22 42353	47.0003335	2	58 95475	-22 42428							0.5									
	2458529.75	258 94984	-22 42499	47 054782	2	58 96456	-22 42469							0.4									
t	2458529.88	258,96956	-22.42647	47.039085	2	258.97128	-22.42517							0.3						_			
F	2458530	258,98906	-22.4279	47.023458	2	58.97744	-22.42585							0.2									0.1
t	2458530.13	259.0085	-22.42924	47.007966	2	58.98575	-22.42685							0.1								• 10	8-
t	2458530.25	259.02801	-22,43052	46.992585		258,9985	-22.42824															Fi	"t"
t	2458530.38	259.0477	-22.43179	46.977221	2	259.01709	-22.43005							a		0.2	0.4		0.5	0.8	1		
F	2458530.5	259.06754	-22.43309	46.961763	2	259.04174	-22.43224																

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Collecting Data

- Horizon's Web-Interface by NASA's Jet Propulsion Laboratory
- Set location and observation intervals
- Collect Julian date, light minutes, right ascension, and declination
- Copy from plain text output
- Paste into pre-made Excel template •

- Determine time in days ullet
- Calculate the angular \bullet separation between Jupiter and each moon using the following equation:

$$\Delta \theta = \sqrt{((\alpha_{\rm M} - \alpha_{\rm J})\cos(\text{radians }\delta_{\rm J}))^2 + (\delta_{\rm M} - \delta_{\rm J})^2 \times \text{sgn}(\Delta \alpha)}$$

 δ = declination and α = right ascension

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1 1	upiter RA	Jupiter DEC	Light Min	lo RA	lo Dec	Time (days)	Δθ°	Fit°	Diff ²										
2	258.54024	-22.43078	35.63622	258.50033	-22.42781	0	-0.03701			ω (rad/sec))	0 <			>	Average Di	istance (m)		
3	258.52865	-22.43019	35.642397	258.48842	-22.42773	0.0833333	-0.03727				_		_						_
4	258.51697	-22.42956	35.648383	258.47996	-22.42784	0.1666667	-0.03425			φ (rad)	_	0 <			>	Amplitude			_
5	258.50525	-22.42889	35.653609	258.47471	-22.42806	0.25	-0.02824			40	_	0 4				Deried (day	(r)	,	_
7	258.49555	-22.42010	35 659841	258.47217	-22.42651	0.3333333	-0.01976			40	_	0				Period (day	(5)		_
8	258,47043	-22.42666	35.660174	258,47205	-22.4286	0.4100007	0.00245			y ²						Period (sec	onds)		-
9	258.45906	-22.42591	35.658467	258.47248	-22.42854	0.5833333	0.01268			^		1				1 21104 (322			_
10	258.44779	-22.42518	35.654809	258.47185	-22.42827	0.6666667	0.02245			Res	set					Jupiter's M	ass (kg)		-
11	258.43656	-22.4245	35.649465	258.46916	-22.42778	0.75	0.03031												
12	258.4253	-22.42386	35.642859	258.46364	-22.42705	0.8333333	0.03558												
13	258.41395	-22.42325	35.635534	258.45471	-22.42608	0.9166667	0.03778												
14	258.40248	-22.42266	35.628102	258.44215	-22.42488	1	0.03674												
15	258.39088	-22.42205	35.621189	258.42605	-22.42347	1.0833333	0.03254												
16	258.37919	-22.42142	35.615375	258.40683	-22.42192	1.1666667	0.02556				_								
17	258.36746	-22.42074	35.611139	258.38518	-22.42027	1.25	0.01639												
18	258.35576	-22.42002	35.608817	258.36203	-22.41862	1.3333333	0.00596				_								
19	258.34414	-22.41926	35.60857	258.33843	-22.41705	1.4166667	-0.00572												
20	258.55265	-22.41849	35.0103/4	258.31544	-22.41566	1.5	-0.01616												
21	258 31001	-22.41/75	35.610162	256.29409	-22.41452	1.0000000	-0.02354												-
23	258 29877	-22.417	35 625297	258 25941	-22 41319	1.0000007	-0.03652				_								
24	258,28751	-22.41566	35.631867	258.24705	-22.41301	1.8333333	-0.0375				_								
25	258.27616	-22.41505	35.638283	258.23816	-22.4131	1.9166667	-0.03518												
26	258.26469	-22.41445	35.643977	258.2325	-22.41337	2	-0.02978												
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Declination and Right Ascension

Angular separation is calculated with the following equation based on Pythagorean theorem:

 $\Delta \theta = \sqrt{((\alpha_{\rm M} - \alpha_{\rm J}) \cos(\text{radians } \delta_{\rm J}))^2 + (\delta_{\rm M} - \delta_{\rm J})^2 \times \text{sgn}(\Delta \alpha)}$

 δ = declination and α = right ascension

- Plot $\Delta \theta$ as a function of time ightarrow
- Use the sliders and graph to ulletestimate the amplitude, phase shift, and angular frequency
- Calculate the fit using the \bullet following equation:

Fit = $\Delta\theta \sin(\omega t + \varphi)$

- Square the difference between the fit and $\Delta \theta$
- Add a second set of data to your graph of fit as a function of time

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1	Jupiter RA	Jupite	er DEC	Light Min	lo RA	lo Dec	Time (days)	Δθ°	Fit®	Diff ²					
2	258.54024	-22	2.43078	35.63622	258.5003	3 -22.42781	0	-0.03701	0.09891	0.018473772	2 ω (rad/sec)	4 <	<		> Average Distance (m)
3	258.52865	-22	2.43019	35.642397	258.4884	2 -22.42773	0.0833333	-0.03727	0.02192	0.003502725	5				
4	258.51697	-22	2.42956	35.648383	258.4799	6 -22.42784	0.1666667	-0.03425	-0.05749	0.000539886	5 ф (rad)	9 ·	<		> Amplitude
5	258.50525	-22	2.42889	35.653609	258.4747	1 -22.42806	0.25	-0.02824	-0.13057	0.010470017	7				
6	258.49355	-22	2.42818	35.657564	258.4721	7 -22.42831	0.3333333	-0.01976	-0.18927	0.028731841	1 40	0.24	<		> Period (days)
7	258.48193	-22	2.42743	35.659841	258.4715	9 -22.42851	0.4166667	-0.00962	-0.22713	0.047313313	3				
8	258.47043	-22	2.42666	35.660174	258.4720	5 -22.4286	0.5	0.00245	-0.24	0.058781213	³ χ ²	0.62985			Period (seconds)
9	258.45906	-22	2.42591	35.658467	258.4724	8 -22.42854	0.5833333	0.01268	-0.22644	0.057178725	Res	et			
10	258.44779	-22	2.42518	35.654809	258.4718	5 -22.42827	0.6666667	0.02245	-0.18795	0.044271721	1				Jupiter's Mass (kg)
11	258.43656	-2	22.4245	35.649465	258.4691	6 -22.42778	0.75	0.03031	-0.12878	0.025309744	4		0.5 1	4 5	
12	258.4253	-22	2.42386	35.642859	258.4636	4 -22.42705	0.8333333	0.03558	-0.05542	0.008282527	703				2
13	258.41395	-22	2.42325	35.635534	258.4547	1 -22.42608	0.9166667	0.03778	0.02403	0.000189162	2				
14	258.40248	-22	2.42266	35.628102	258.4421	5 -22.42488	1	0.03674	0.10084	0.004109091	1 0.2		• • •	\sim	
15	258.39088	-22	2.42205	35.621189	258.4260	5 -22.42347	1.0833333	0.03254	0.16655	0.017957674	4			/ • \	
16	258.37919	-22	2.42142	35.615375	258.4068	3 -22.42192	1.1666667	0.02556	0.21392	0.035481968	0.1		• /	• \	
17	258.36746	-22	2.42074	35.611139	258.3851	8 -22.42027	1.25	0.01639	0.23775	0.048999592	2 \		•	. \	
18	258.35576	-22	2.42002	35.608817	258.3620	3 -22.41862	1.3333333	0.00596	0.2354	0.052640092	2 o	\setminus	• /	\	
19	258.34414	-22	2.41926	35.60857	258.3384	3 -22.41705	1.4166667	-0.00572	0.20713	0.045308006	5 o	\mathbf{X}	05 1	• 15	2
20	258.33265	-22	2.41849	35.610374	258.3154	4 -22.41566	1.5	-0.01616	0.15607	0.029662534	-0.1				
21	258.32128	-22	2.41773	35.614026	258.2940	9 -22.41452	1.5833333	-0.02534	0.08782	0.012805662	2				
22	258.31001	-	-22.417	35.619162	258.275	2 -22.41369	1.66666667	-0.03235	0.00991	0.001/85/11	-0.2			•	
23	258.29877	-24	2.41631	35.625297	258.2594	1 -22.41319	1./5	-0.03652	-0.0691	0.001061288		•	\smile		•••
24	258.28/51	-24	2.41500	35.031807	258.2470	5 -22.41301	1.85555555	-0.03/5	-0.1405	0.010608729	-0.3				
25	258.27010	-24	2.41505	25.638283	258.2581	5 -22.4131 5 -22.41227	1.9100007	-0.03518	-0.19045	0.025999720					
20	236.20409	-24	2.41445	35.043977	236.232	5 -22.41557	2	-0.02978	-0.25074	0.040364023	2				
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- χ^2 is the sum of the difference column
- Run the Excel Solver add-in to minimize the χ^2 value by changing the amplitude, phase shift, and angular frequency variables

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1	Jupiter RA	Jupiter DEC	Light Min	lo RA	lo Dec	Time (days)	Δθ°	Fit°	Diff ²							
2	258.54024	-22.43078	35.63622	258.50033	-22.42781	0	-0.03701	-0.03727	6.7743E-08	ω (rad/sec)	3.54039	<			> Averag	e Distance (m)
3	258.52865	-22.43019	35.642397	258.48842	-22.42773	0.0833333	-0.03727	-0.03735	7.40783E-09			_				
4	258.51697	-22.42956	35.648383	258.47996	-22.42784	0.1666667	-0.03425	-0.03421	1.92492E-09	φ (rad)	17.1237	<			> Amplit	ude
5	258.50525	-22.42889	35.653609	258.47471	-22.42806	0.2222222	-0.02824	-0.02811	1./6631E-08	40	0.02773	1			> Deried	(days)
7	258,49555	-22.42818	25 6508/1	258,47217	-22.42851	0.0000000	-0.01976	-0.01958	5.3/160E-08	70	0.05772	`			Period	(aays)
8	258 47043	-22.42745	35 660174	258,47205	-22.42031	0.4100007	0.00302	0.00930	6.06464E-07	v ²	1 SE-06				Period	(seconds)
9	258 45906	-22.42000	35 658467	258 47248	-22.42854	0 5833333	0.00245	0.00107	1 51846E-08	٨	1.50-00				renou	(seconds)
10	258,44779	-22.42518	35.654809	258.47185	-22.42827	0.66666667	0.02245	0.02236	9.23615E-09	Rese	et				Jupiter	's Mass (kg)
11	258,43656	-22,4245	35.649465	258,46916	-22.42778	0.75	0.03031	0.03023	7.46155E-09							(
12	258.4253	-22.42386	35.642859	258.46364	-22.42705	0.8333333	0.03558	0.03548	1.02802E-08	0		0.5	1	1.5		2
13	258.41395	-22.42325	35.635534	258.45471	-22.42608	0.9166667	0.03778	0.03767	1.23788E-08	0.05						
14	258.40248	-22.42266	35.628102	258.44215	-22.42488	1	0.03674	0.03661	1.69425E-08	0.04		-	-			
15	258.39088	-22.42205	35.621189	258.42605	-22.42347	1.0833333	0.03254	0.03238	2.66193E-08	0.03						
16	258.37919	-22.42142	35.615375	258.40683	-22.42192	1.1666667	0.02556	0.02535	4.12974E-08	0.02						
17	258.36746	-22.42074	35.611139	258.38518	-22.42027	1.25	0.01639	0.01613	6.38275E-08	0.01		/				
18	258.35576	-22.42002	35.608817	258.36203	-22.41862	1.3333333	0.00596	0.00552	1.93583E-07	0		/				
19	258.34414	-22.41926	35.60857	258.33843	-22.41705	1.4166667	-0.00572	-0.00557	2.43629E-08	0		0.5	1	15		2
20	258.33265	-22.41849	35.610374	258.31544	-22.41566	1.5	-0.01616	-0.01617	2.34979E-10	-0.01						
21	258.32128	-22.41773	35.614026	258.29409	-22.41452	1.5833333	-0.02534	-0.02538	2.06337E-09	-0.02	/					
22	258.31001	-22.417	35.619162	258.2752	-22.41369	1.6666667	-0.03235	-0.0324	2.72842E-09	-0.03					~ ~	/ _
23	258.29877	-22.41631	35.625297	258.25941	-22.41319	1.75	-0.03652	-0.03662	9.79108E-09	-0.04 🦰						•
24	258.28/51	-22.41566	35.631867	258.24705	-22.41301	1.85555533	-0.03/5	-0.03/6/	5.03115E-08	-0.05						
25	258.2/616	-22.41505	35.638283	258.23816	-22.4131	1.910000/	-0.03518	-0.03547	8.116/E-08							
20	200.20409	-22.41443	33.043977	200.2020	-22.4100/	2	-0.02978	-0.0302	1.70496E-07							
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• Calculate the average distance (meters), amplitude, and period (seconds)

 1.8×10^{10} meters Avg Distance = Avg Light Minutes $\times -$ 1 Light Minute

> $\Delta\theta \times \pi \times \text{Average Distance}$ Amplitude = -180

Period (seconds) =
$$\frac{2\pi}{\omega} \times 24 \times 3600$$

Using Kepler's 3rd Law of ulletplanetary motion, calculate the mass of Jupiter

$$\frac{a^3}{p^2} = \frac{GM}{4\pi^2}$$

M =mass of the parent body a =amplitude in meters p = period of the orbit in seconds

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1	Jupiter	RA	Jupit	er DEC	Lig	ght M	in
2	258.	54024	-2	2.43078		35.63	622
3	258.	52865	-2	2.43019	3	5.642	397
4	258.	51697	-2	2.42956	3	5.648	383
5	258.	50525	-2	2.42889	3	5.653	609
6	258.	49355	-2	2.42818	3	5.657	564
7	258.	48193	-2	2.42743	3	5.659	841
8	258.	47043	-2	2.42666	3	5.660)174
9	258.	45906	-2	2.42591	3	5.658	8467
10	258.	44779	-2	2.42518	3	5.654	809
11	258.	43656	-	22.4245	3	5.649	465
12	258	8.4253	-2	2.42386	3	5.642	859
13	258.	41395	-2	2.42325	3	5.635	534
14	258.	40248	-2	2.42266	3	5.628	3102
15	258.	39088	-2	2.42205	3	5.621	189
16	258.	37919	-2	2.42142	3	5.615	375
17	258.	36746	-2	2.42074	3	5.611	139
18	258.	35576	-2	2.42002	3	5.608	817
19	258.	34414	-2	2.41926		35.60	857
20	258.	33265	-2	2.41849	3	5.610	374
21	258.	32128	-2	2.41773	3	5.614	026
22	258.	31001		-22.417	3	5.619	9162
23	258.	29877	-2	2.41631	3	5.625	297
24	258.	28751	-2	2.41566	3	5.631	867
25	258.	27616	-2	2.41505	3	5.638	3283
26	258.	26469	-2	2.41445	3	5.643	977
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lo RA	lo Dec	Time (days)	Δθ°	Fit°	Diff ²							
258.50033	-22.42781	0	-0.03701	-0.03727	6.7743E-08	ω (rad/se	ec) 3.540	89 <			>	Average Distance (m)
258.48842	-22.42773	0.0833333	-0.03727	-0.03735	7.40783E-09							
258.47996	-22.42784	0.1666667	-0.03425	-0.03421	1.92492E-09	φ (rad)	17.12	87 <	(>	Amplitude
258.47471	-22.42806	0.25	-0.02824	-0.02811	1.76631E-08							
258.47217	-22.42831	0.3333333	-0.01976	-0.01958	3.37186E-08	Δθ	0.037	2 <			>	Period (days)
258.47159	-22.42851	0.4166667	-0.00962	-0.00936	6.79075E-08							
258.47205	-22.4286	0.5	0.00245	0.00167	6.06464E-07	χ ^z	1.5E-	06				Period (seconds)
258.47248	-22.42854	0.5833333	0.01268	0.01256	1.51846E-08	B	Reset					
258.47185	-22.42827	0.6666667	0.02245	0.02236	9.23615E-09		(CSCI					Jupiter's Mass (kg)
258.46916	-22.42778	0.75	0.03031	0.03023	7.46155E-09							
258.46364	-22.42705	0.8333333	0.03558	0.03548	1.02802E-08	0.05	0		0.5 1	1 1	.5	2
258.45471	-22.42608	0.9166667	0.03778	0.03767	1.23788E-08	0.05						
258.44215	-22.42488	1	0.03674	0.03661	1.69425E-08	0.04			-			
258.42605	-22.42347	1.0833333	0.03254	0.03238	2.66193E-08	0.03						
258.40683	-22.42192	1.1666667	0.02556	0.02535	4.12974E-08	0.02						
258.38518	-22.42027	1.25	0.01639	0.01613	6.38275E-08	0.01			_			
258.36203	-22.41862	1.3333333	0.00596	0.00552	1.93583E-07							
258.33843	-22.41705	1.4166667	-0.00572	-0.00557	2.43629E-08	0	0		05	1 1	5	2
258.31544	-22.41566	1.5	-0.01616	-0.01617	2.34979E-10	-0.01		1				
258.29409	-22.41452	1.5833333	-0.02534	-0.02538	2.06337E-09	-0.02		/				
258.2752	-22.41369	1.6666667	-0.03235	-0.0324	2.72842E-09	-0.03						
258.25941	-22.41319	1.75	-0.03652	-0.03662	9.79108E-09	-0.04						• •
258.24705	-22.41301	1.8333333	-0.0375	-0.03767	3.03115E-08	0.05						
258.23816	-22.4131	1.9166667	-0.03518	-0.03547	8.1167E-08	-0.05						
258.2325	-22.41337	2	-0.02978	-0.0302	1.78498E-07							
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Future Research

Apply this method to calculating the mass of other planets

Develop an alternative procedure for CLEA's "Jupiter's Moons and the Speed of Light" exercise

"The first astrophysicist and the last scientific astrologer."

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