KEPLER'S 3RD LAW OF PLANETARY MOTION

Implementing New Web Capabilities to Develop Updated Laboratory Procedures

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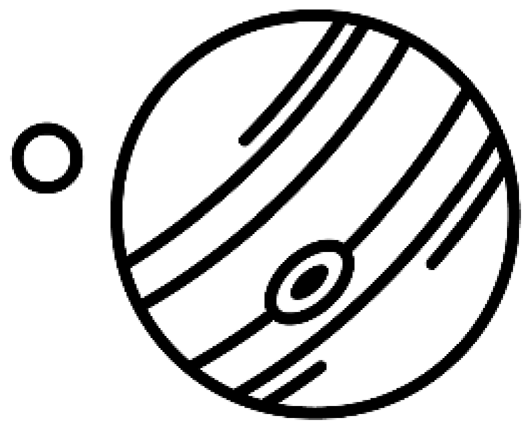


Goal: Create a revised implementation of the Moons of Jupiter lab using new web capabilities, specifically, Horizons Web-Interface.

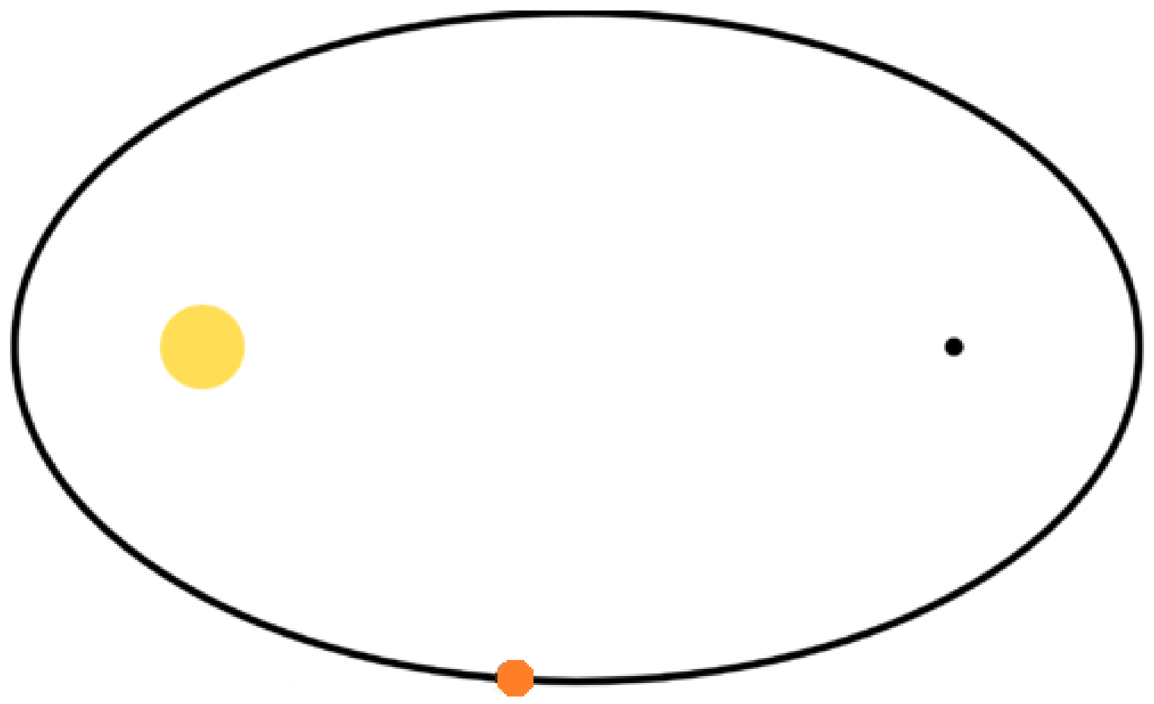
Johannes Kepler

- German astronomer and mathematician
- 1571-1630
- Believed there was an inherent order to the universe and that “those laws [of nature] are within the grasp of the human mind…”
- Work Included: Rudolphine Tables, The Kepler Conjecture, and Kepler’s Laws of Planetary Motion

Kepler's Laws of Planetary Motion

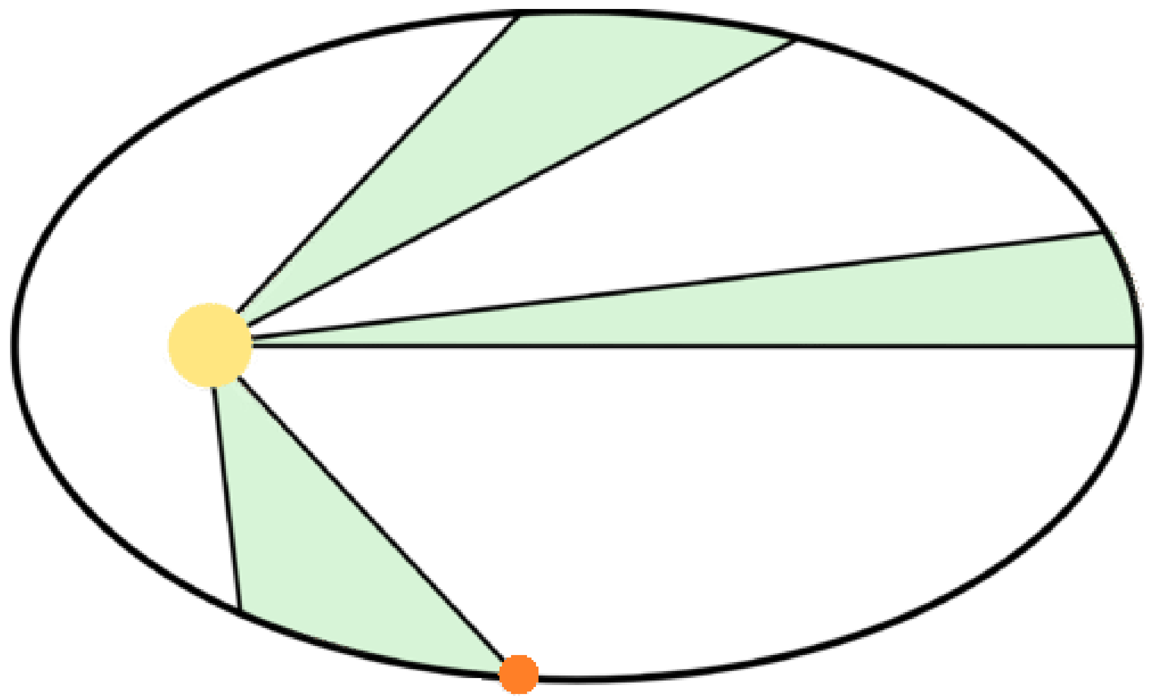
1

The orbit of every planet is an ellipse with the Sun at one of the two foci



2

A line joining a planet and the Sun sweeps out equal areas during equal intervals of time



3

The square of the orbital period of a planet is directly proportional to the cube of the semi-major axis of its orbit

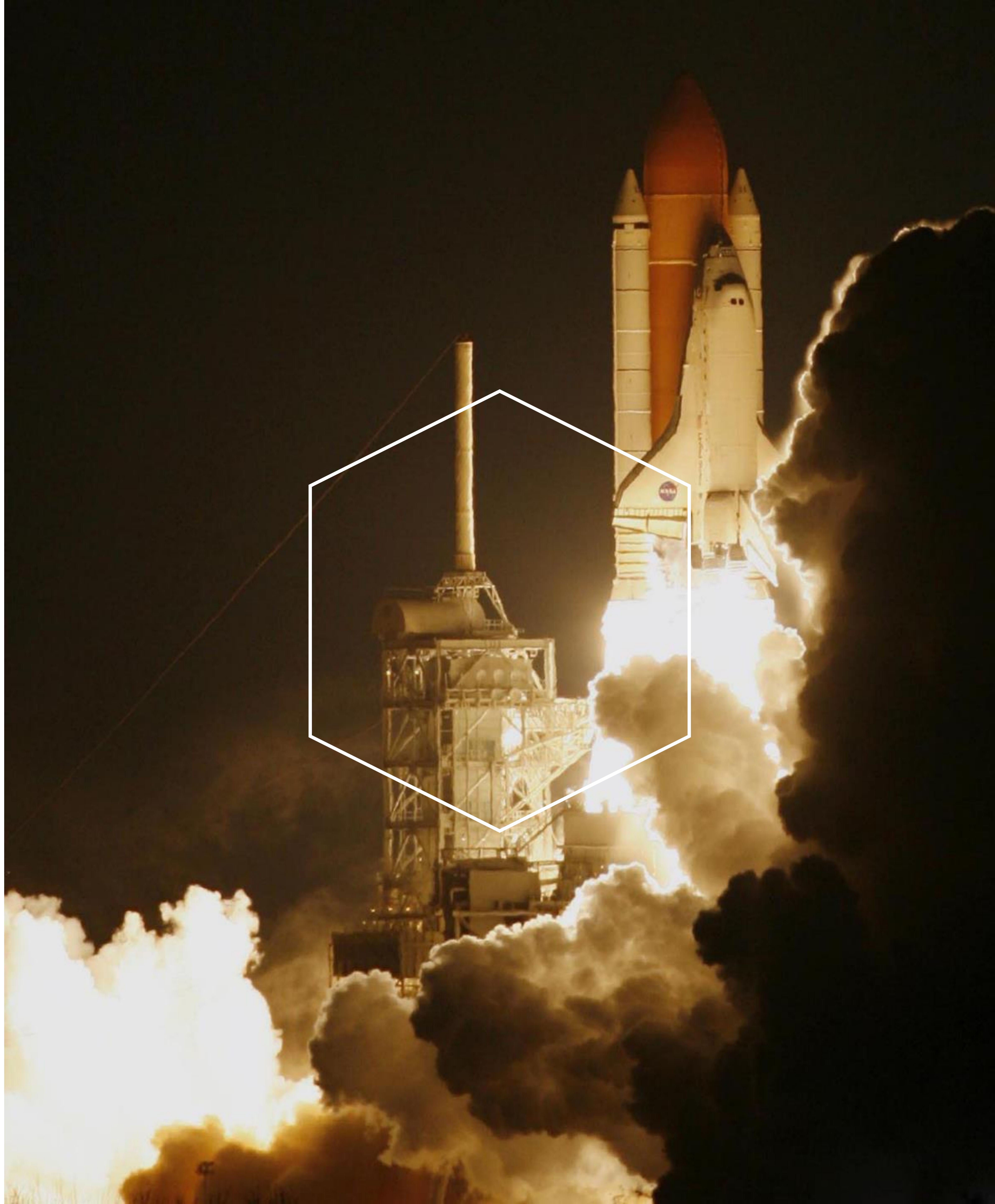
$$\left(\frac{T_1}{T_2}\right)^2 = \left(\frac{a_1}{a_2}\right)^3$$

T = period

a = semi − major axis

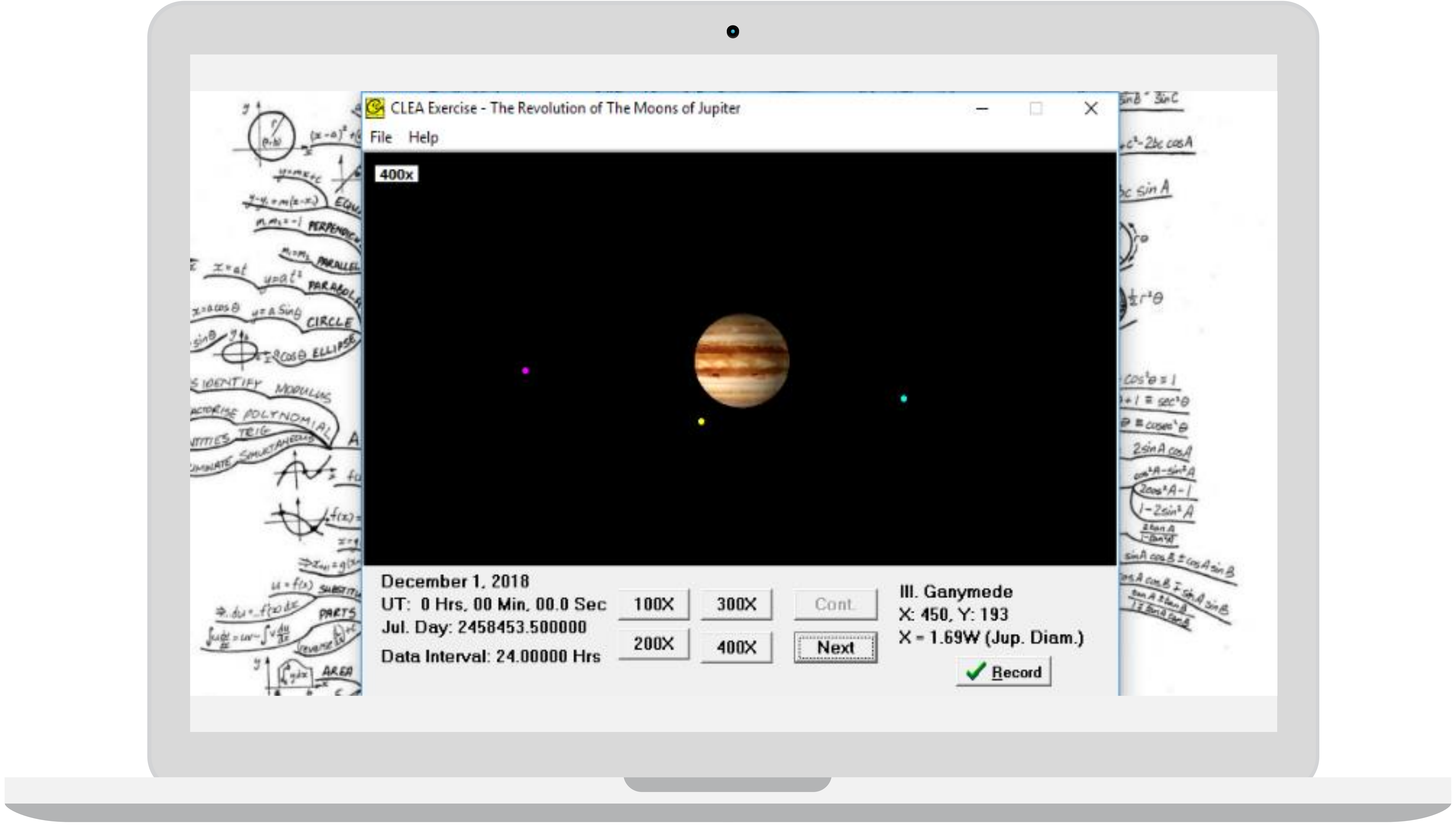
The Significance of Kepler's Laws

- Supported the model of the Sun as the center of the Solar System
- Inspired Isaac Newton to explain motion with the Law of Universal Gravitation



Contemporary Laboratory Exercises in Astronomy

- Designed by Gettysburg College in the 1990s
- A resource of free software-based laboratory exercises for astronomy students
- Still available for download but no longer being updated



Collect the Data

In the CLEA software, students record the position of the four moons in relation to Jupiter

Analyze the Data

-Graph the position versus time

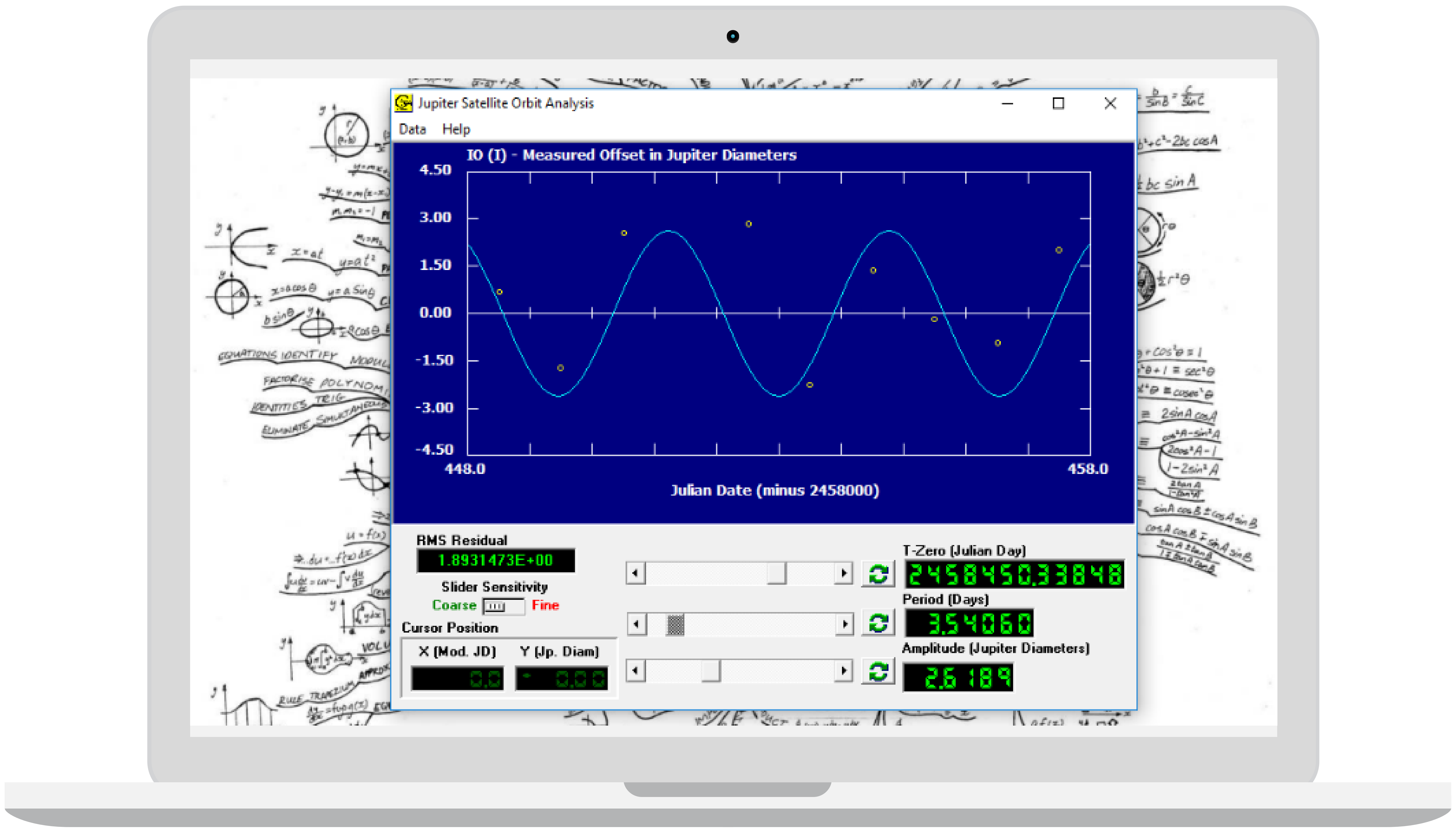
-Fit this data to a sine function

-Record period and amplitude

Calculate the Mass of Jupiter

Use the following equation to calculate the mass of Jupiter:

$$\mathrm{M} = \frac{a^3}{p^2}$$



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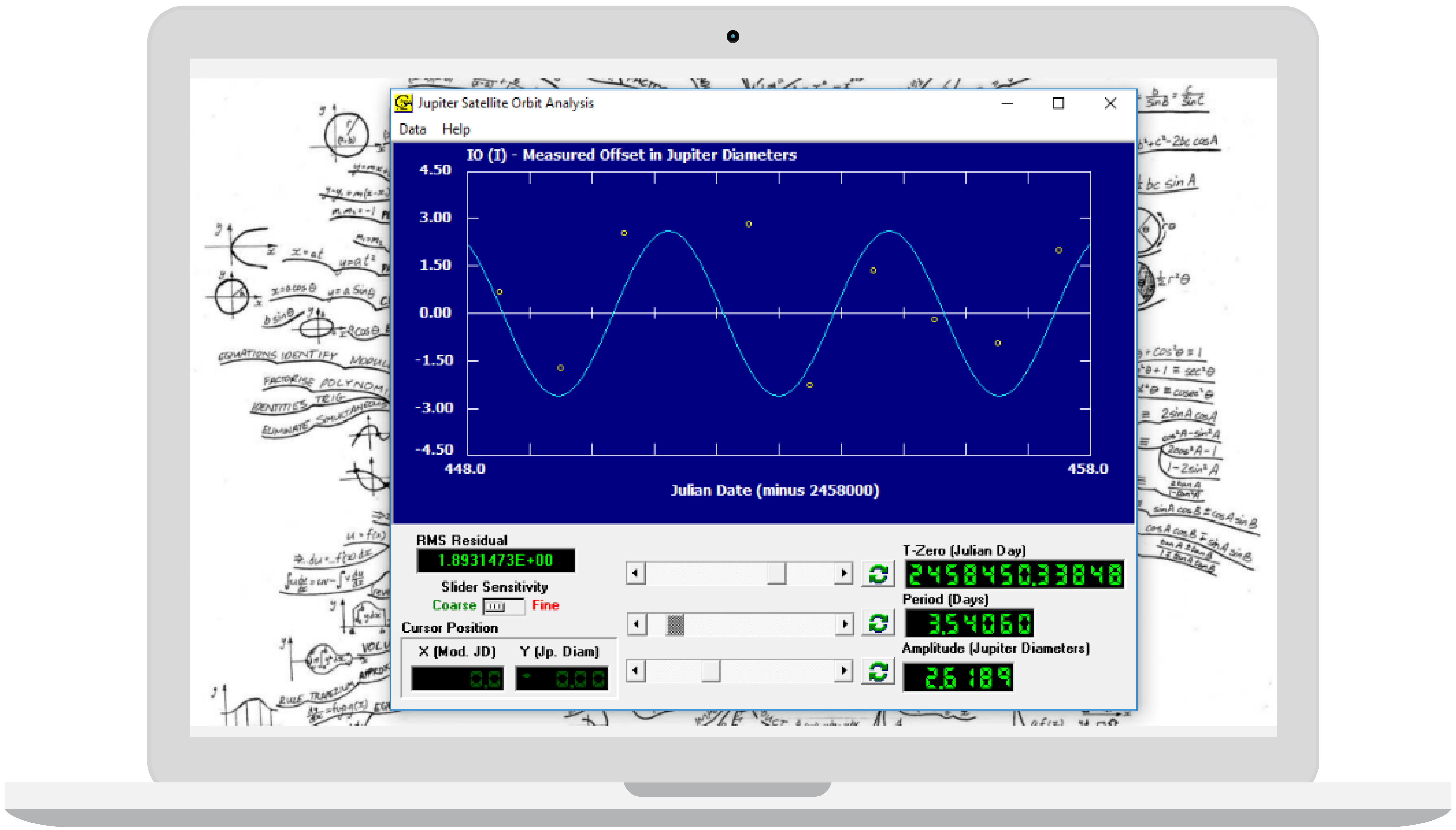

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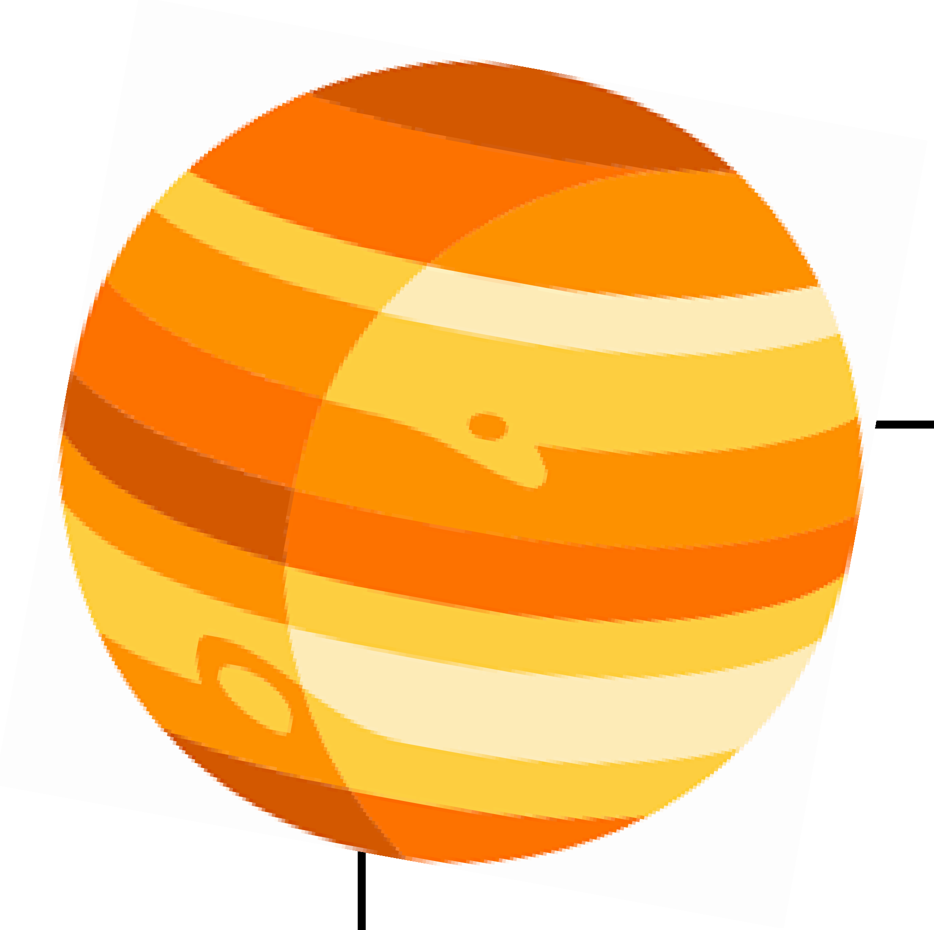
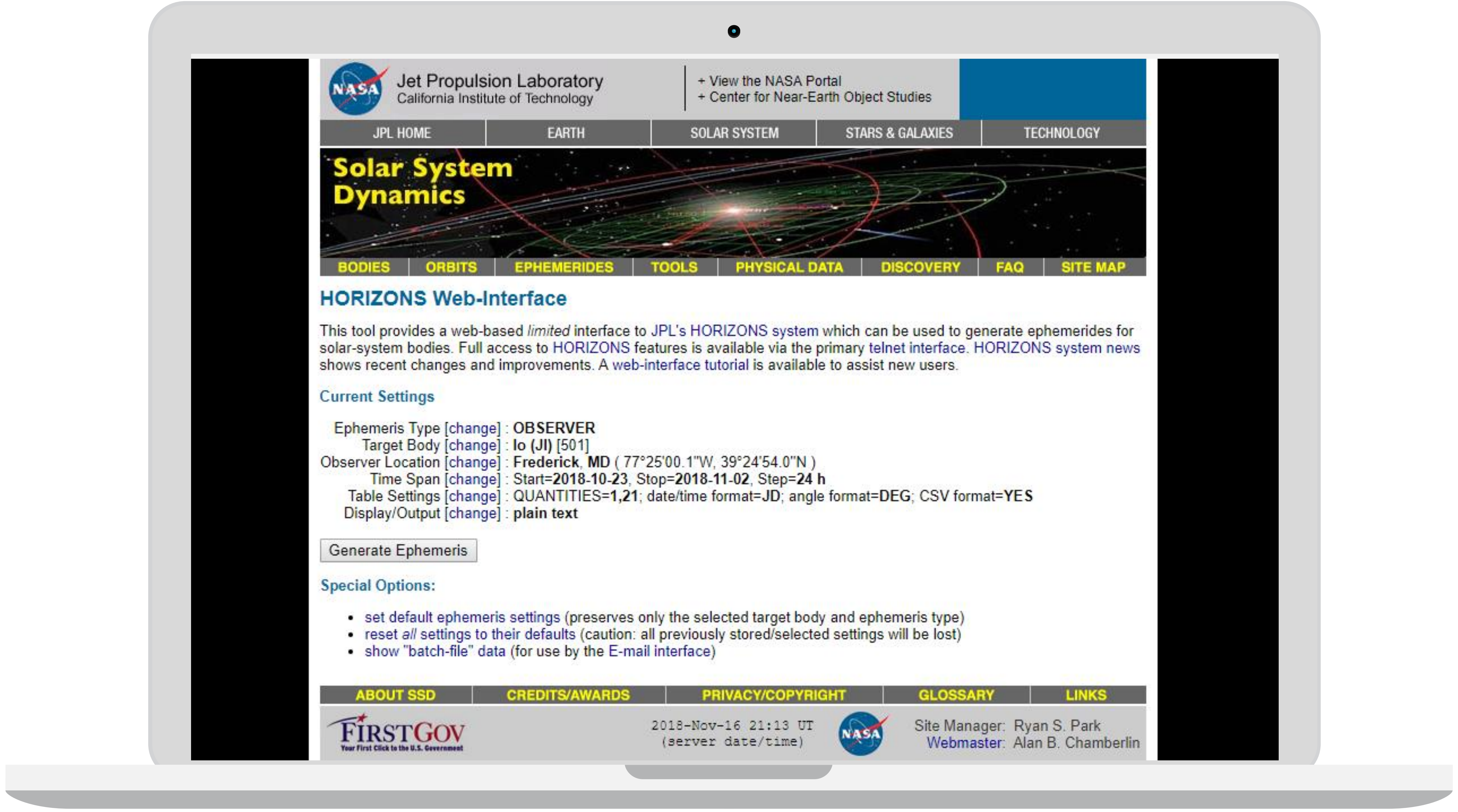


An Updated Approach...



Collecting Data

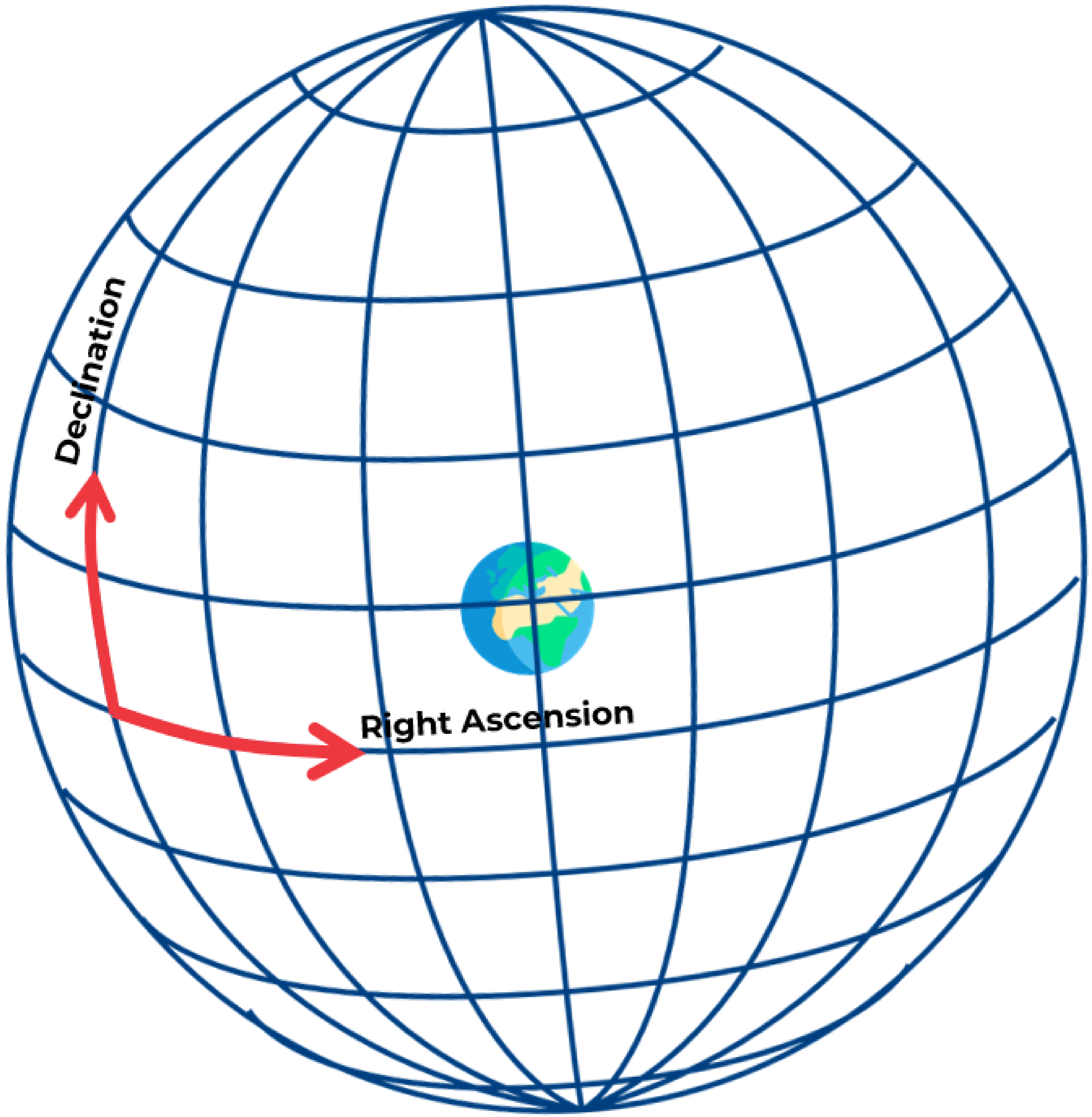
- Horizon's Web-Interface by NASA's Jet Propulsion Laboratory
- Set location and observation intervals
- Collect Julian date, light minutes, right ascension, and declination

Declination and Right Ascension

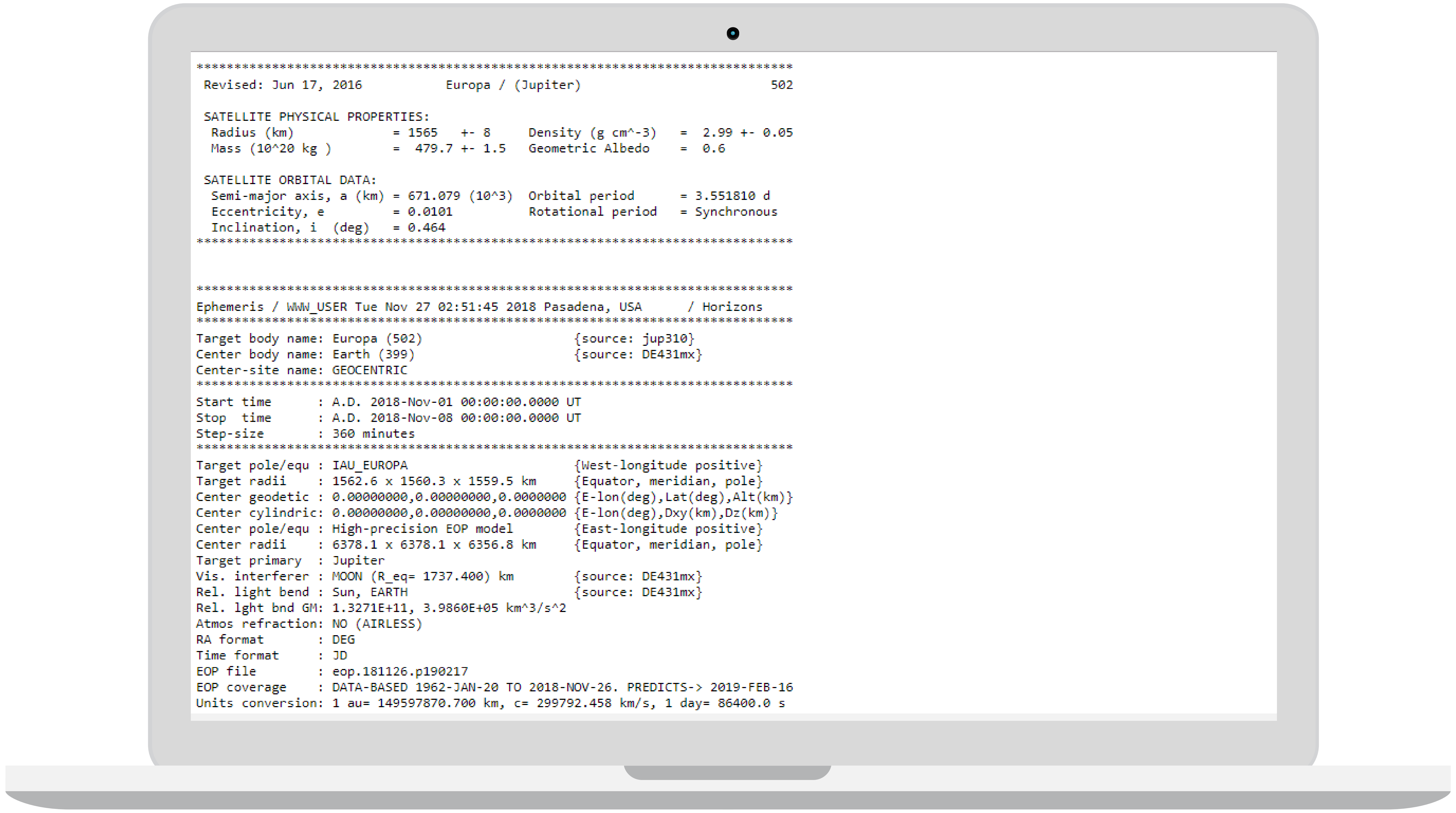
Celestial Sphere
- No set axis
- Centered about Earth
- Relates the position of objects by measuring their passage

Right Ascension
- "Longitude"
- Measured in hours with 24 hours being 1 rotation

Declination
- "Latitude"
- Celestial equator: declination of 0˚
- North celestial pole: declination of +90˚
- South celestial pole: declination of –90˚



Collecting Data



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*******************************************************************************
 Revised: Jun 17, 2016            Europa / (Jupiter)                      502

 SATELLITE PHYSICAL PROPERTIES:
  Radius (km)             = 1565   +- 8     Density (g cm^-3)   =  2.99 +- 0.05
  Mass (10^20 kg )        =  479.7 +- 1.5   Geometric Albedo    =  0.6

 SATELLITE ORBITAL DATA:
  Semi-major axis, a (km) = 671.079 (10^3)  Orbital period      = 3.551810 d
  Eccentricity, e         = 0.0101          Rotational period   = Synchronous
  Inclination, i  (deg)   = 0.464
*******************************************************************************


*******************************************************************************
Ephemeris / WWW_USER Tue Nov 27 02:51:45 2018 Pasadena, USA      / Horizons
*******************************************************************************
Target body name: Europa (502)                    {source: jup310}
Center body name: Earth (399)                     {source: DE431mx}
Center-site name: GEOCENTRIC
*******************************************************************************
Start time      : A.D. 2018-Nov-01 00:00:00.0000 UT
Stop  time      : A.D. 2018-Nov-08 00:00:00.0000 UT
Step-size       : 360 minutes
*******************************************************************************
Target pole/equ : IAU_EUROPA                      {West-longitude positive}
Target radii    : 1562.6 x 1560.3 x 1559.5 km     {Equator, meridian, pole}
Center geodetic : 0.00000000,0.00000000,0.0000000 {E-lon(deg),Lat(deg),Alt(km)}
Center cylindric: 0.00000000,0.00000000,0.0000000 {E-lon(deg),Dxy(km),Dz(km)}
Center pole/equ : High-precision EOP model        {East-longitude positive}
Center radii    : 6378.1 x 6378.1 x 6356.8 km     {Equator, meridian, pole}
Target primary  : Jupiter
Vis. interferer : MOON (R_eq= 1737.400) km        {source: DE431mx}
Rel. light bend : Sun, EARTH                      {source: DE431mx}
Rel. lght bnd GM: 1.3271E+11, 3.9860E+05 km^3/s^2
Atmos refraction: NO (AIRLESS)
RA format       : DEG
Time format     : JD
EOP file        : eop.181126.p190217
EOP coverage    : DATA-BASED 1962-JAN-20 TO 2018-NOV-26. PREDICTS-> 2019-FEB-16
Units conversion: 1 au= 149597870.700 km, c= 299792.458 km/s, 1 day= 86400.0 s
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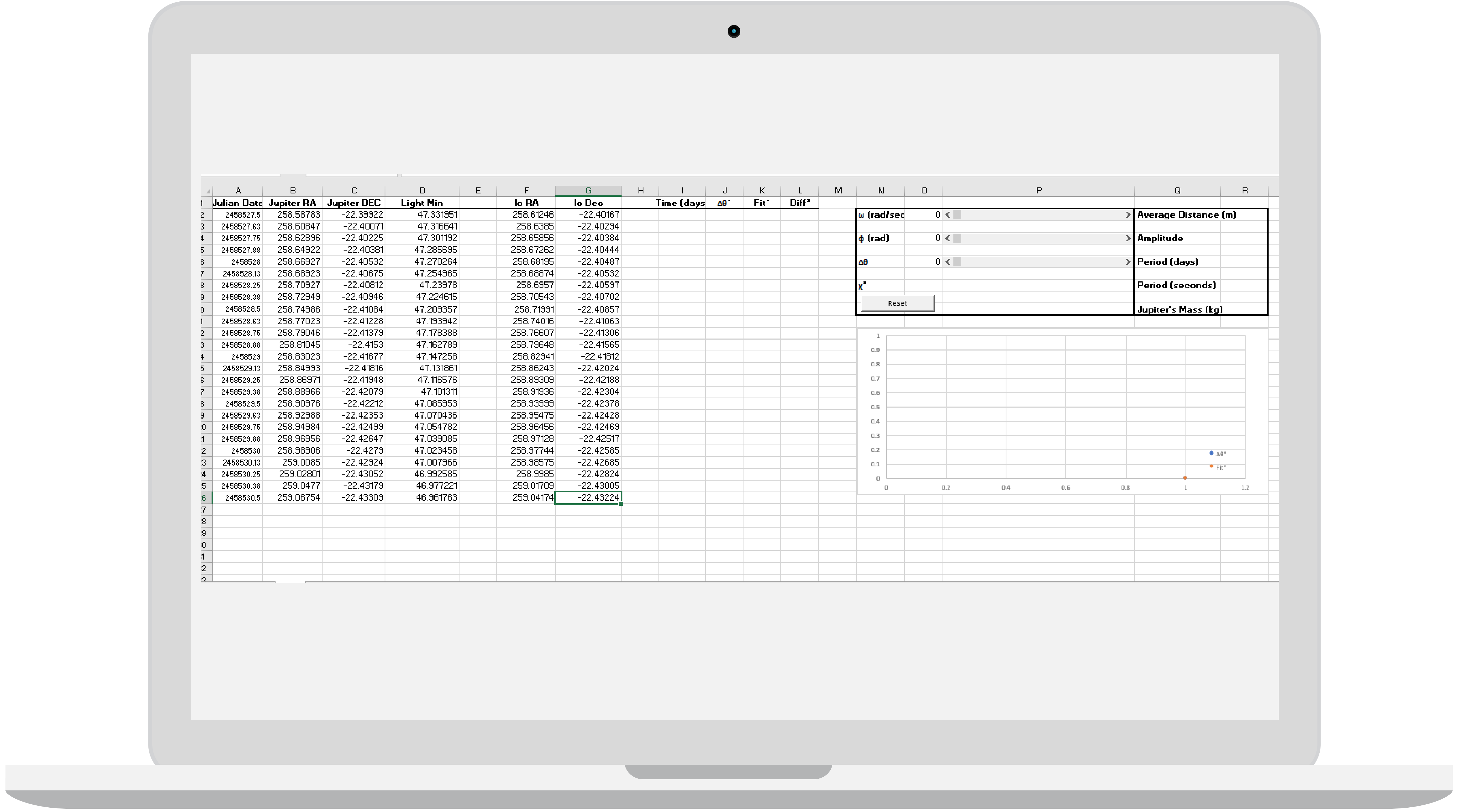
- Horizon’s Web-Interface by NASA’s Jet Propulsion Laboratory
- Set location and observation intervals
- Collect Julian date, light minutes, right ascension, and declination
- Copy from plain text output



Collecting Data

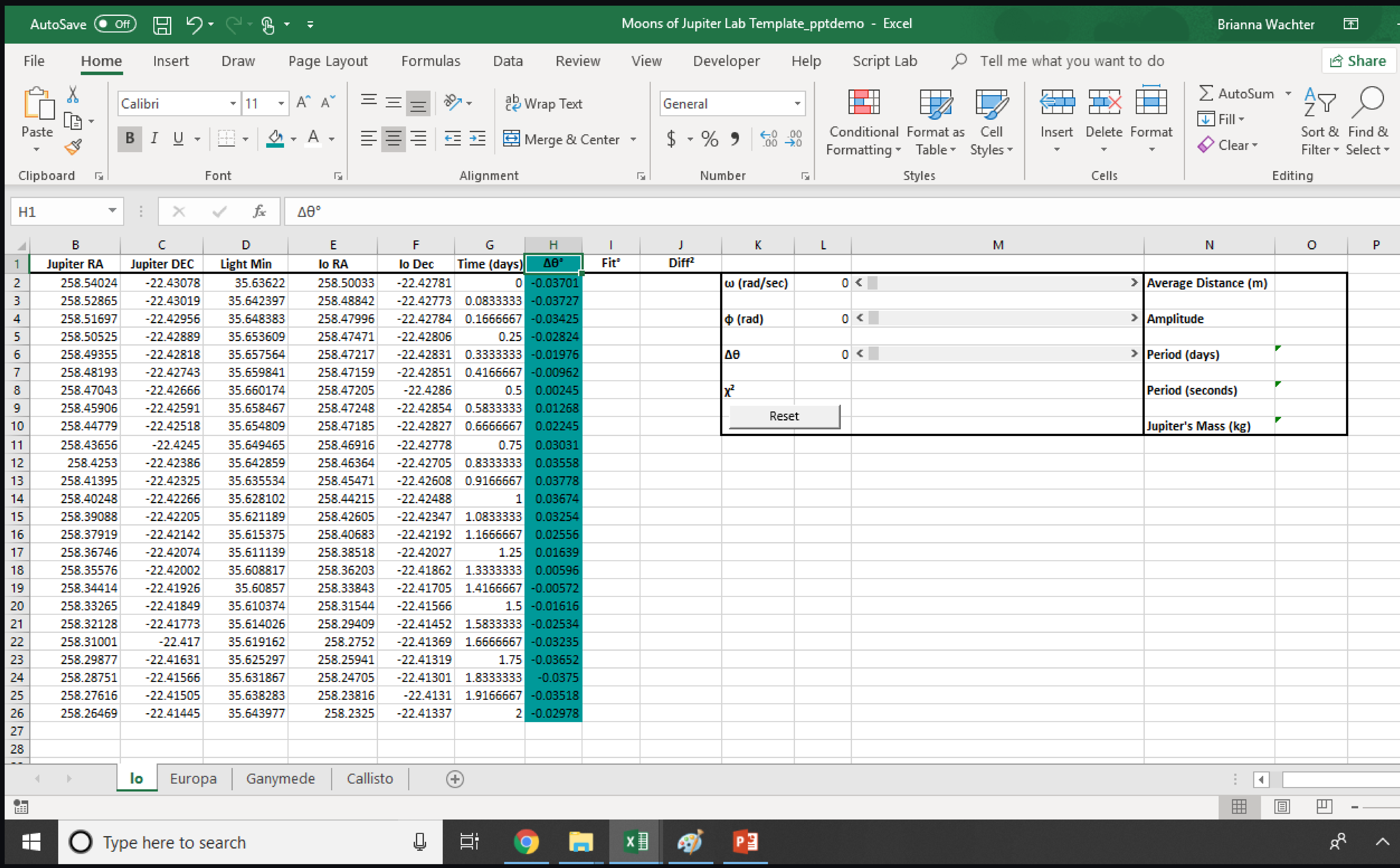
- Horizon's Web-Interface by NASA's Jet Propulsion Laboratory
- Set location and observation intervals
- Collect Julian date, light minutes, right ascension, and declination
- Copy from plain text output
- Paste into pre-made Excel template

Analyzing the Data

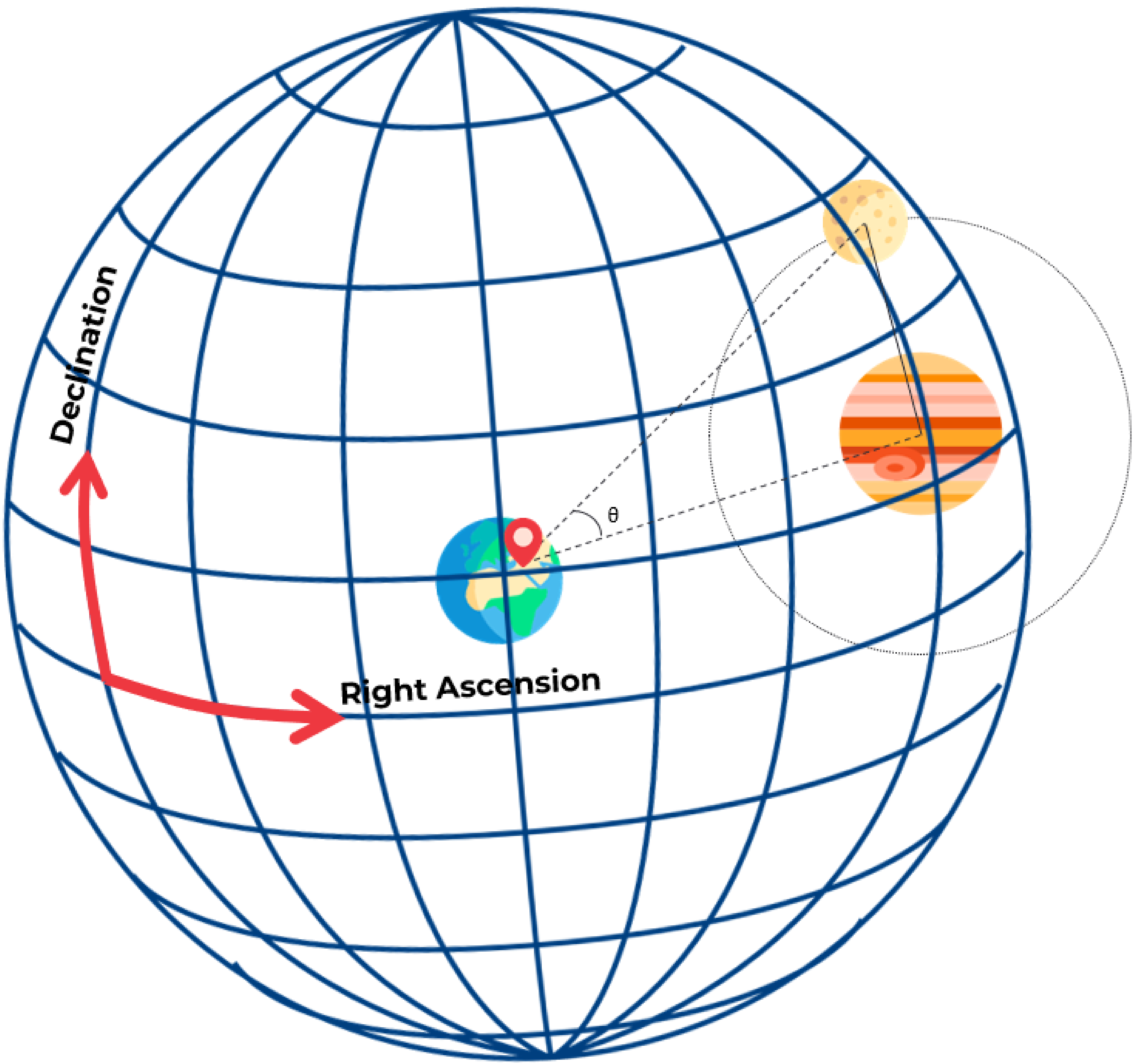
- Determine time in days
- Calculate the angular separation between Jupiter and each moon using the following equation:

$$\Delta\theta = \sqrt{((\alpha_M - \alpha_J)\cos(\text{radians}\ \delta_J))^2 + (\delta_M - \delta_J)^2} \times \text{sgn}(\Delta\alpha)$$

$$\delta = \text{declination and}\ \alpha = \text{right ascension}$$



Jupiter RA	Jupiter DEC	Light Min	Io RA	Io Dec	Time (days)	Δθ°	Fit°	Diff²
258.54024	-22.43078	35.63622	258.50033	-22.42781	0	-0.03701		
258.52865	-22.43019	35.642397	258.48842	-22.42773	0.0833333	-0.03727		
258.51697	-22.42956	35.648383	258.47996	-22.42784	0.1666667	-0.03425		
258.50525	-22.42889	35.653609	258.47471	-22.42806	0.25	-0.02824		
258.49355	-22.42818	35.657564	258.47217	-22.42831	0.3333333	-0.01976		
258.48193	-22.42743	35.659841	258.47159	-22.42851	0.4166667	-0.00962		
258.47043	-22.42666	35.660174	258.47205	-22.4286	0.5	0.00245		
258.45906	-22.42591	35.658467	258.47248	-22.42854	0.5833333	0.01268		
258.44779	-22.42518	35.654809	258.47185	-22.42827	0.6666667	0.02245		
258.43656	-22.4245	35.649465	258.46916	-22.42778	0.75	0.03031		
258.4253	-22.42386	35.642859	258.46364	-22.42705	0.8333333	0.03558		
258.41395	-22.42325	35.635534	258.45471	-22.42608	0.9166667	0.03778		
258.40248	-22.42266	35.628102	258.44215	-22.42488	1	0.03674		
258.39088	-22.42205	35.621189	258.42605	-22.42347	1.0833333	0.03254		
258.37919	-22.42142	35.615375	258.40683	-22.42192	1.1666667	0.02556		
258.36746	-22.42074	35.611139	258.38518	-22.42027	1.25	0.01639		
258.35576	-22.42002	35.608817	258.36203	-22.41862	1.3333333	0.00596		
258.34414	-22.41926	35.60857	258.33843	-22.41705	1.4166667	-0.00572		
258.33265	-22.41849	35.610374	258.31544	-22.41566	1.5	-0.01616		
258.32128	-22.41773	35.614026	258.29409	-22.41452	1.5833333	-0.02534		
258.31001	-22.417	35.619162	258.2752	-22.41369	1.6666667	-0.03235		
258.29877	-22.41631	35.625297	258.25941	-22.41319	1.75	-0.03652		
258.28751	-22.41566	35.631867	258.24705	-22.41301	1.8333333	-0.0375		
258.27616	-22.41505	35.638283	258.23816	-22.4131	1.9166667	-0.03518		
258.26469	-22.41445	35.643977	258.2325	-22.41337	2	-0.02978		



Declination and Right Ascension

Angular separation is calculated with the following equation based on Pythagorean theorem:

$$\Delta\theta = \sqrt{\left(\left(\alpha_{\mathrm{M}} - \alpha_{\mathrm{J}}\right)\cos\left(\mathrm{radians}\ \delta_{\mathrm{J}}\right)\right)^2 + \left(\delta_{\mathrm{M}} - \delta_{\mathrm{J}}\right)^2} \times \mathrm{sgn}(\Delta\alpha)$$

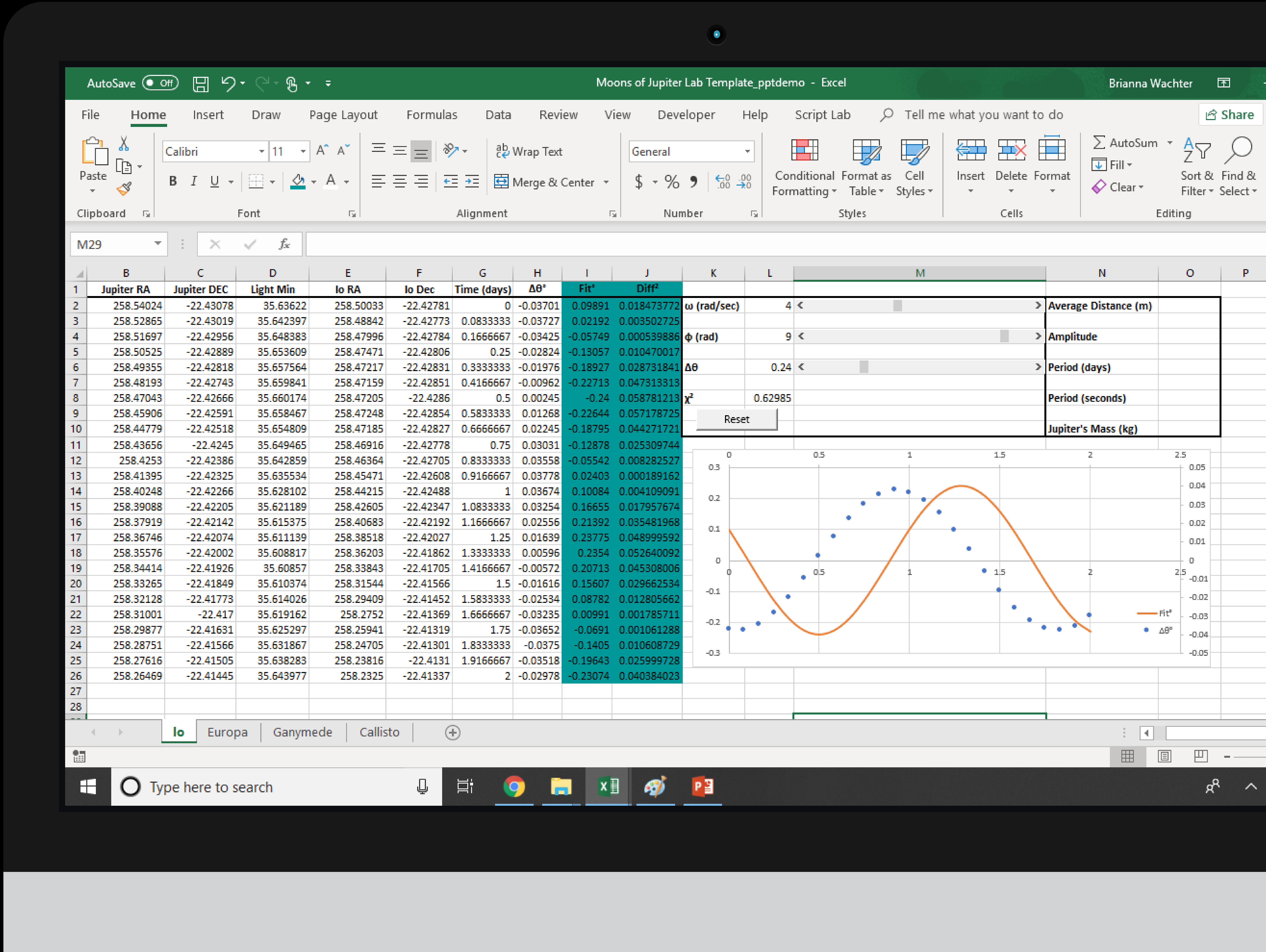
$$\delta \ = \ \text{declination and } \alpha = \ \text{right ascension}$$

Analyzing the Data

- Plot $\Delta\theta$ as a function of time
- Use the sliders and graph to estimate the amplitude, phase shift, and angular frequency
- Calculate the fit using the following equation:

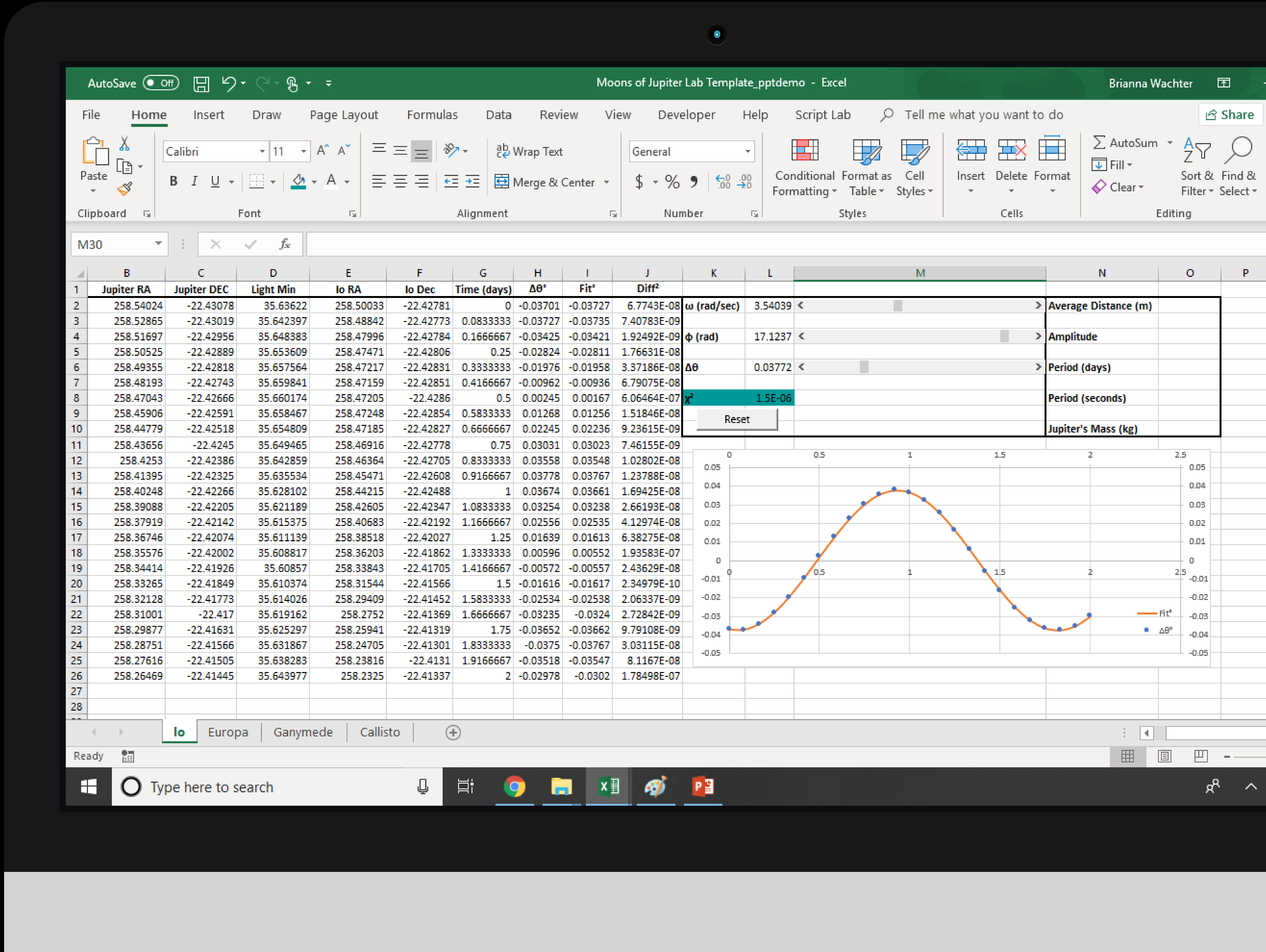
$$\text{Fit} = \Delta\theta \sin(\omega t + \varphi)$$

- Square the difference between the fit and $\Delta\theta$
- Add a second set of data to your graph of fit as a function of time



Analyzing the Data

- χ^2 is the sum of the difference column
- Run the Excel Solver add-in to minimize the χ^2 value by changing the amplitude, phase shift, and angular frequency variables



Analyzing the Data

- Calculate the average distance (meters), amplitude, and period (seconds)

$$\text{Avg Distance} = \text{Avg Light Minutes} \times \frac{1.8 \times 10^{10}\ \text{meters}}{1\ \text{Light Minute}}$$

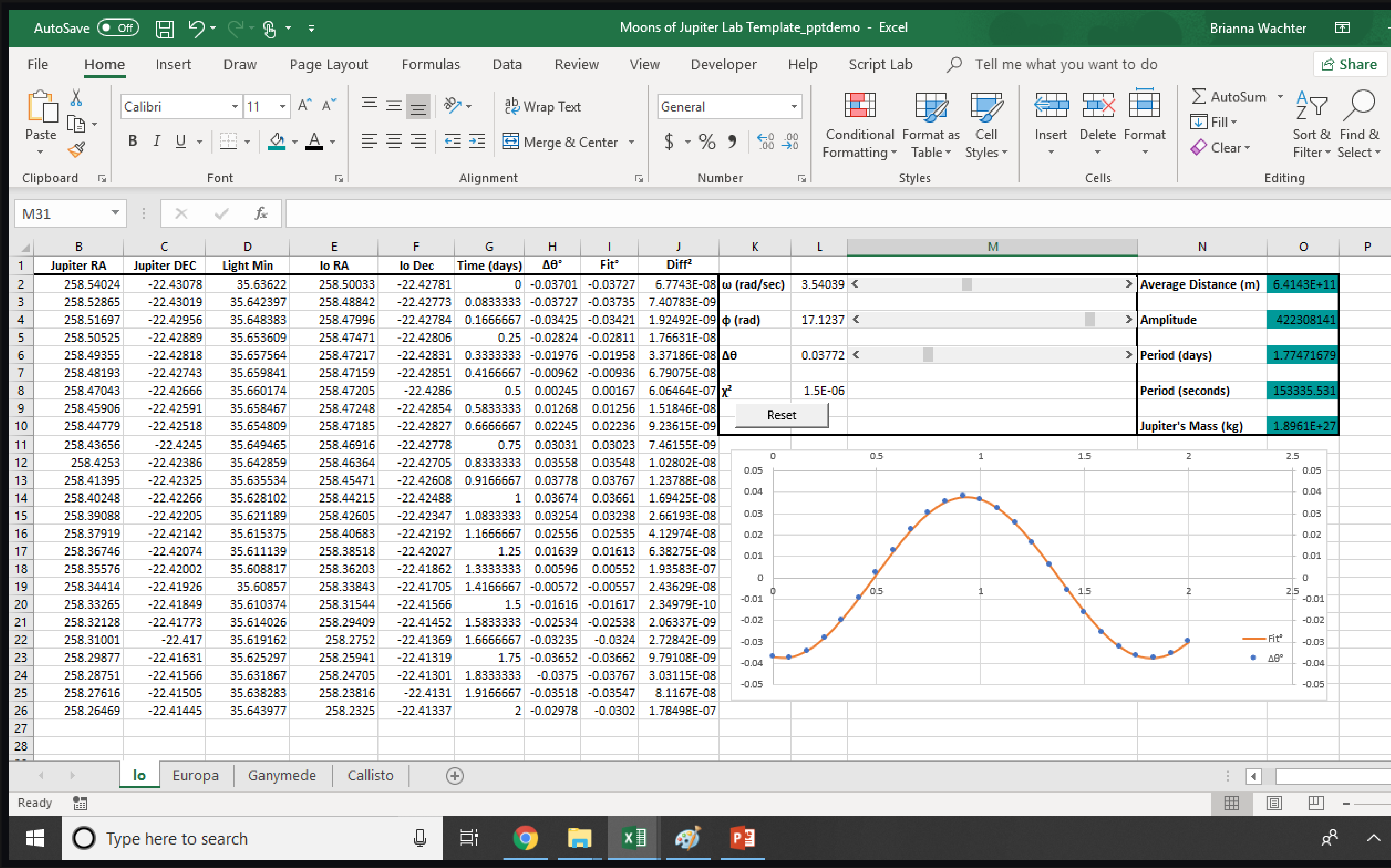
$$\text{Amplitude} = \frac{\Delta\theta \times \pi \times \text{Average Distance}}{180}$$

$$\text{Period (seconds)} = \frac{2\pi}{\omega} \times 24 \times 3600$$

- Using Kepler's 3rd Law of planetary motion, calculate the mass of Jupiter

$$\frac{a^3}{p^2} = \frac{GM}{4\pi^2}$$

$M =$ mass of the parent body
$a =$ amplitude in meters
$p =$ period of the orbit in seconds



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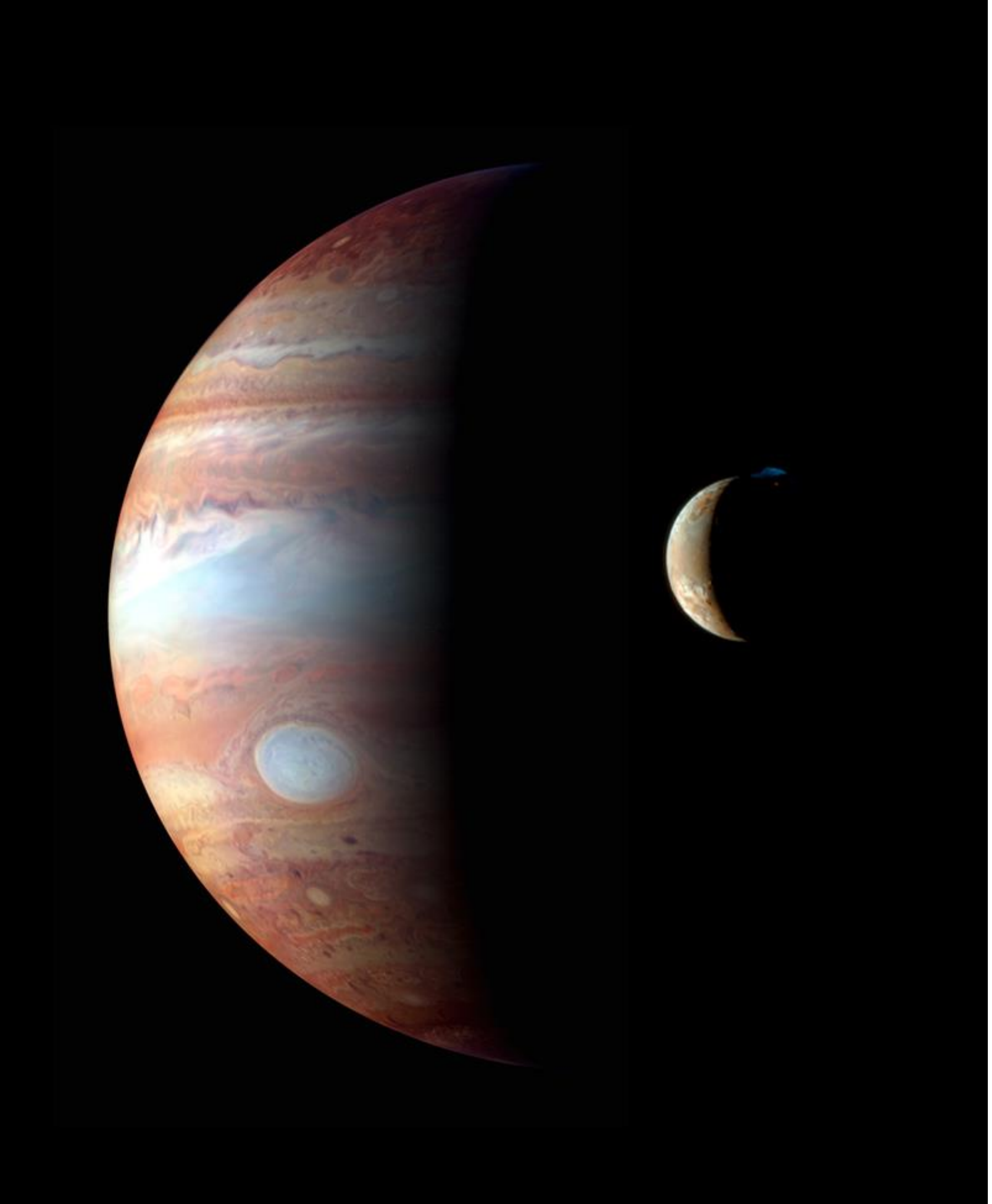
2	258.54024	-22.43078	35.63622	258.50033	-22.42781	0	-0.03701	-0.03727	6.7743E-08
3	258.52865	-22.43019	35.642397	258.48842	-22.42773	0.0833333	-0.03727	-0.03735	7.40783E-09
4	258.51697	-22.42956	35.648383	258.47996	-22.42784	0.1666667	-0.03425	-0.03421	1.92492E-09
5	258.50525	-22.42889	35.653609	258.47471	-22.42806	0.25	-0.02824	-0.02811	1.76631E-08
6	258.49355	-22.42818	35.657564	258.47217	-22.42831	0.3333333	-0.01976	-0.01958	3.37186E-08
7	258.48193	-22.42743	35.659841	258.47159	-22.42851	0.4166667	-0.00962	-0.00936	6.79075E-08
8	258.47043	-22.42666	35.660174	258.47205	-22.4286	0.5	0.00245	0.00167	6.06464E-07
9	258.45906	-22.42591	35.658467	258.47248	-22.42854	0.5833333	0.01268	0.01256	1.51846E-08
10	258.44779	-22.42518	35.654809	258.47185	-22.42827	0.6666667	0.02245	0.02236	9.23615E-09
11	258.43656	-22.4245	35.649465	258.46916	-22.42778	0.75	0.03031	0.03023	7.46155E-09
12	258.4253	-22.42386	35.642859	258.46364	-22.42705	0.8333333	0.03558	0.03548	1.02802E-08
13	258.41395	-22.42325	35.635534	258.45471	-22.42608	0.9166667	0.03778	0.03767	1.23788E-08
14	258.40248	-22.42266	35.628102	258.44215	-22.42488	1	0.03674	0.03661	1.69425E-08
15	258.39088	-22.42205	35.621189	258.42605	-22.42347	1.0833333	0.03254	0.03238	2.66193E-08
16	258.37919	-22.42142	35.615375	258.40683	-22.42192	1.1666667	0.02556	0.02535	4.12974E-08
17	258.36746	-22.42074	35.611139	258.38518	-22.42027	1.25	0.01639	0.01613	6.38275E-08
18	258.35576	-22.42002	35.608817	258.36203	-22.41862	1.3333333	0.00596	0.00552	1.93583E-07
19	258.34414	-22.41926	35.60857	258.33843	-22.41705	1.4166667	-0.00572	-0.00557	2.43629E-08
20	258.33265	-22.41849	35.610374	258.31544	-22.41566	1.5	-0.01616	-0.01617	2.34979E-10
21	258.32128	-22.41773	35.614026	258.29409	-22.41452	1.5833333	-0.02534	-0.02538	2.06337E-09
22	258.31001	-22.417	35.619162	258.2752	-22.41369	1.6666667	-0.03235	-0.0324	2.72842E-09
23	258.29877	-22.41631	35.625297	258.25941	-22.41319	1.75	-0.03652	-0.03662	9.79108E-09
24	258.28751	-22.41566	35.631867	258.24705	-22.41301	1.8333333	-0.0375	-0.03767	3.03115E-08
25	258.27616	-22.41505	35.638283	258.23816	-22.4131	1.9166667	-0.03518	-0.03547	8.1167E-08
26	258.26469	-22.41445	35.643977	258.2325	-22.41337	2	-0.02978	-0.0302	1.78498E-07

ω (rad/sec)	3.54039
ϕ (rad)	17.1237
Δθ	0.03772
χ^2	1.5E-06

Average Distance (m)	6.4143E+11
Amplitude	422308141
Period (days)	1.77471679
Period (seconds)	153335.531
Jupiter's Mass (kg)	1.8961E+27



Future Research

- Apply this method to calculating the mass of other planets
- Develop an alternative procedure for CLEA's "Jupiter's Moons and the Speed of Light" exercise


“The first astrophysicist and
the last scientific astrologer.”

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Kepler Portrait: https://foroparalelo.com/ocio-y-cultura/postea-asigno-cientifico-680219/index2.html
Kepler's 1st and 2nd Law (*modified*): https://dr282zn36sxxg.cloudfront.net/datastreams/f-d%3A4bd04a2a3b11005b1c99c562b00533fce981bf29719de3e09d57afa7%2BIMAGE_THUMB_POSTCARD_TINY%2BIMAGE_THUMB_POSTCARD_TINY.1

References:

CLEA: http://www3.gettysburg.edu/~marschal/clea/CLEAbase.html
Celestial sphere: https://en.wikipedia.org/wiki/Celestial_sphere
RA and DEC: https://en.wikipedia.org/wiki/Right_ascension
Jupiter and Io: https://solarsystem.nasa.gov/resources/803/jupiter-and-io/?category=planets_jupiter
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Clouds of Jupiter: https://solarsystem.nasa.gov/resources/902/chaotic-clouds-of-jupiter/?category=planets_jupiter