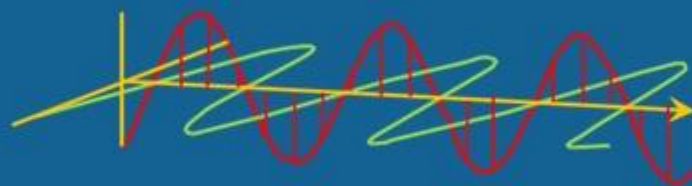
Riding on a Light Beam



Relativistic Doppler Shift And Measuring *ct, x*

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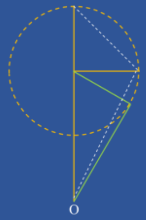
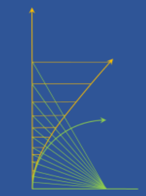
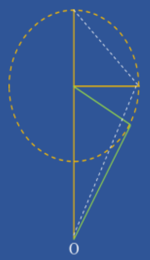


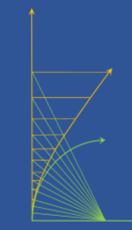
AGENDA



- EVENTS, OBSERVATIONS & MEASUREMENTS
    - Event=Measurement at $v<<c$
    - Galilean Transform
- THE LORENTZ TRANSFORM
    - Minkowski & Brehme Diagrams
    - Buried Assumption: Event=Measurement
- VELOCITY TRIANGLE
    - Observation & Measurement
    - Brehme Angle Beta
    - Worldline vs. Timeline
    - Rotated Reference Frames, Translating at *c*.
- LIGHT BETWEEN REFERENCE FRAMES
    - Tangent to Observer's Timeline
    - Role of Doppler Shift
    - Simultaneous with Stationary Objects, Doppler shift = 1
- MULTIPLE REFERENCE FRAMES AND THE INVARIANT $c\tau$



EVENTS, OBSERVATIONS & MEASUREMENTS

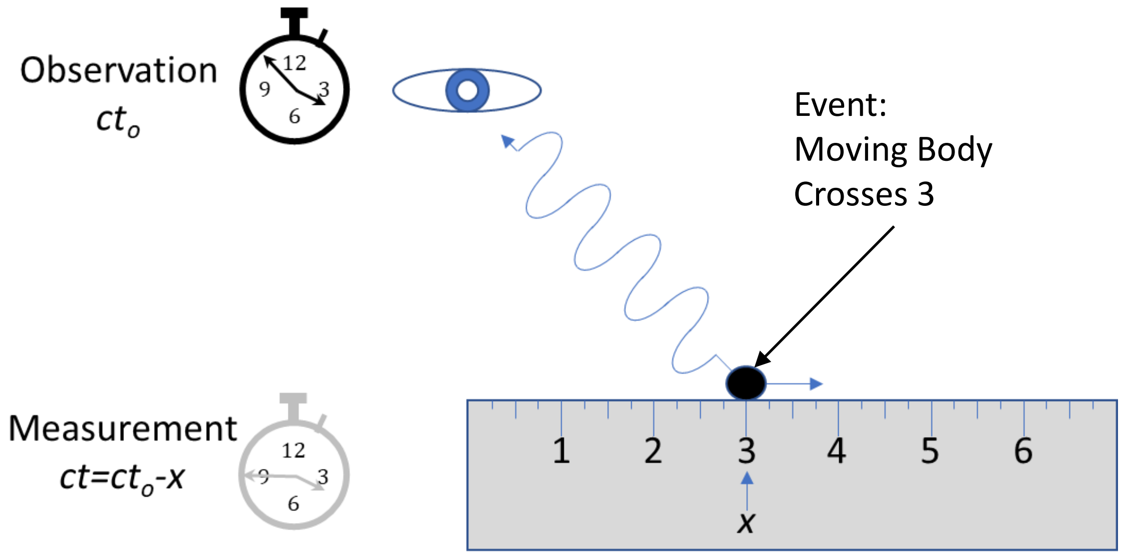


Definition

- NORMALLY NEGLIGIBLY SEPARATED IN TIME OR SPACE FOR $v << c$: NOT TRUE FOR HIGHER *v!*
- EVENT: Proper Event with Zero Spatial Coordinates that Generates Light (Time of Transmission)
- OBSERVATION: Proper Time (Zero Spatial Coordinates) of Receipt of Light from an Event in One Reference Frame By an Observer in Another Reference Frame (Time of Receipt)
- MEASUREMENT: Determination of ct, x Coordinates in Observer's Reference Frame.

EVENTS, OBSERVATIONS & MEASUREMENTS

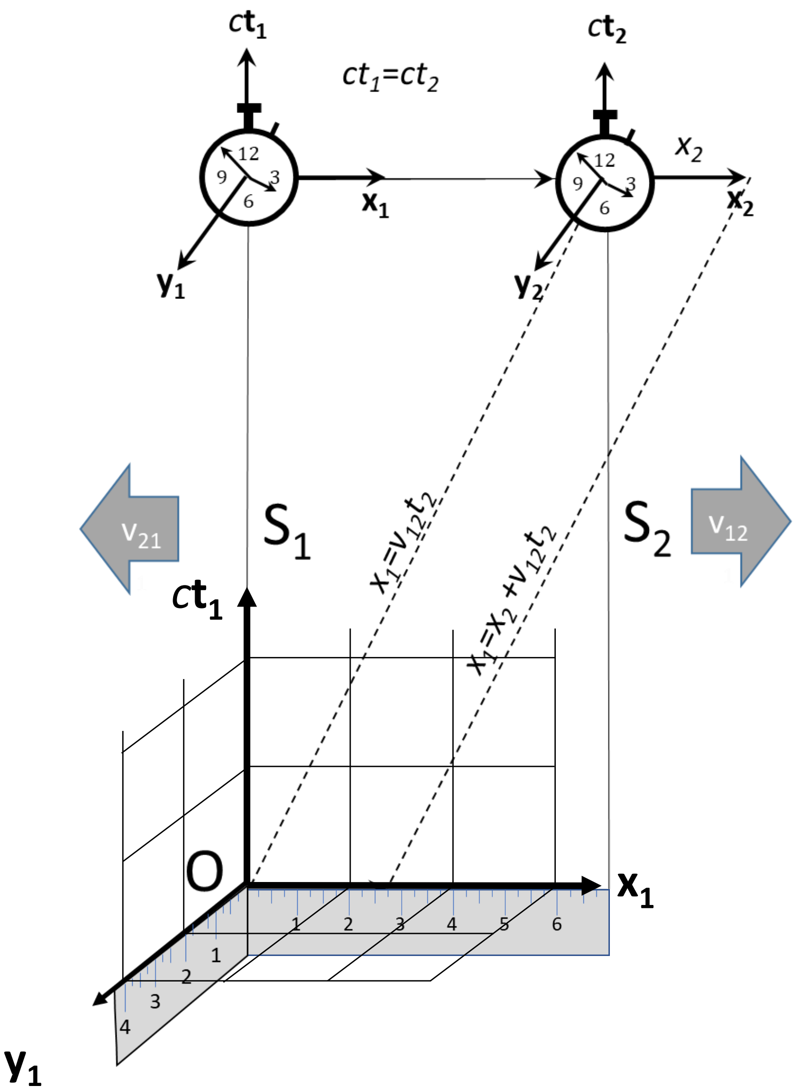
Low Speed Solution



- Observer Observes Light from Body at ct_o Simultaneous with Light from x
- Body was "at" x at Time $ct=ct_o-x$
- Works for Low Speed, But Not High Speed Due to Doppler

EVENTS, OBSERVATIONS & MEASUREMENTS

Galilean Transform



$$x_1 = x_2 + v_{12} \cdot ct_1$$
$$ct_1 = ct_2$$

Transit Time from Event
to Measurement,
Doppler Shift Both Negligible

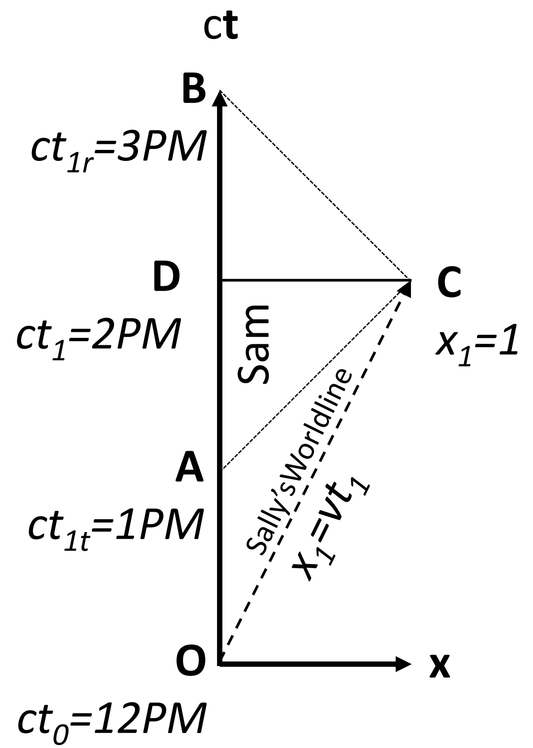
WHAT TIME IS IT, SALLY?

$v=0.5c$

- SALLY LEAVES EARTH AT O
  - $ct_0 = 12\text{PM}$
  - $v/c = 0.5c$

- SAM TRACKS SALLY BY RADAR ON HER WORLDLINE OC
- AT A, SAM ASKS SALLY WHAT TIME IT IS
- AT B SAM GETS HER REPLY AND LOCATES HER AT C BY RADAR EQUATION:
  - $x_1 = (ct_{1r} - ct_{1t})/2 \quad = 1$ *light hour*
  - $ct = (ct_{1r} + ct_{1t})/2 \quad = 2\text{PM}$
  - $v = x/ct = 0.5$

- *BUT SALLY SAID HER CLOCK READ 1:43PM WHEN SHE GOT HIS REQUEST!*



THE LORENTZ TRANSFORMATION

- SIMPLE ENOUGH! TRANSFORMS MEASUREMENT FROM ONE REFERENCE FRAME TO ANOTHER

$$x_2 = \frac{x_1 + \left(\frac{v}{c}\right) ct_1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$ct_2 = \frac{ct_1 + \left(\frac{v}{c}\right) x_1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

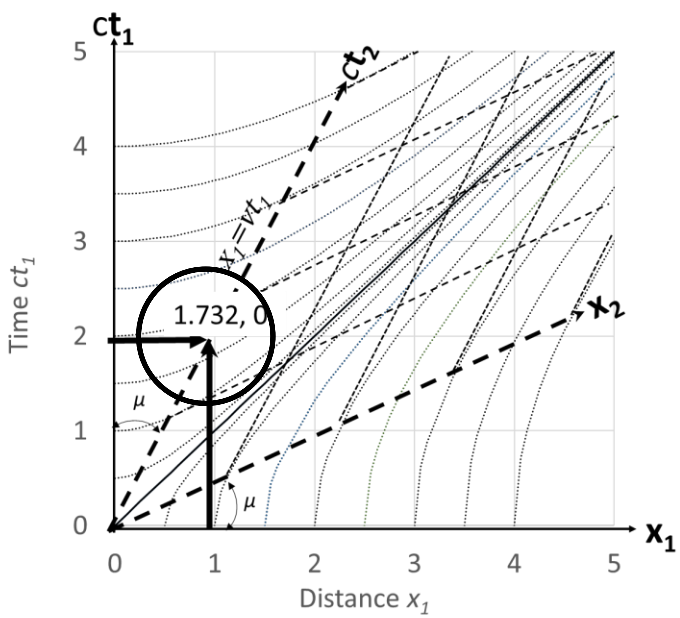
- FOR $v << c$, $x << ct$, BECOMES THE GALILEAN TRANSFORM:

$$x_2 \cong x_1 + vt_1$$
$$ct_2 \cong ct_1$$

- BUT WHAT DOES IT MEAN?

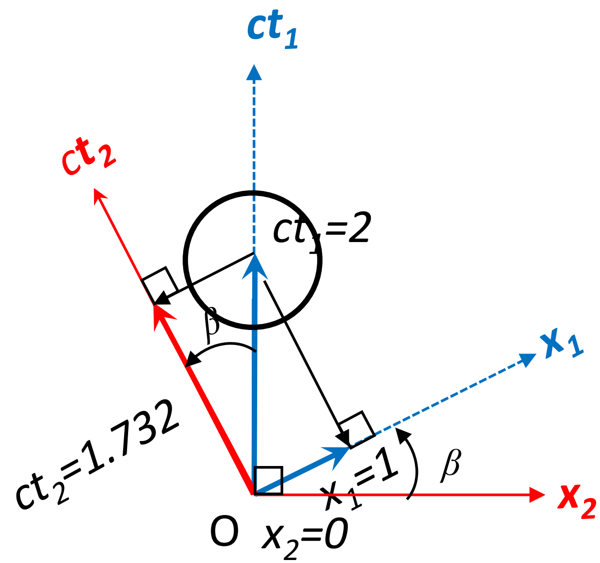
LORENTZ TRANSFORMATION

Minkowski and Brehme Diagrams



Minkowski Diagram

World Line *x=vt* and Constant $c\tau = \sqrt{(ct_1)^2 - x_1^2}$



Brehme Diagram

Reference Frame Orthogonalities

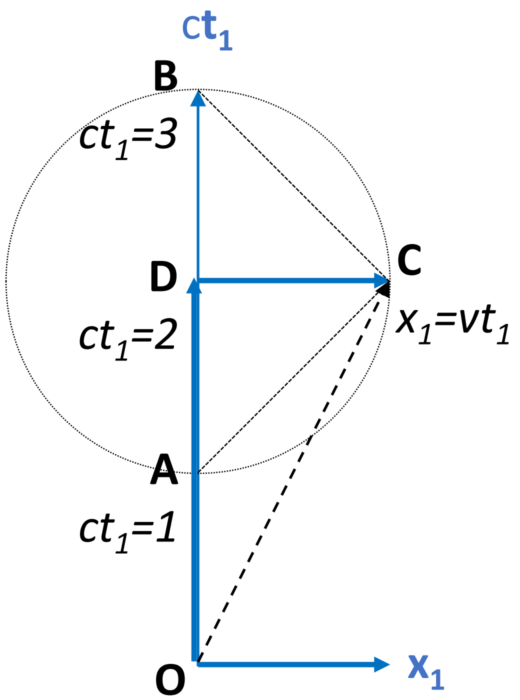
Expanded/Reduced by Brehme Angle B=sin^{-1}(v/c)

- ASSUMPTION: EMITTER'S EVENT (1.732,0) AND THE OBSERVER'S MEASUREMENT (2,1) ARE THE SAME POINT
- THEREFORE, THE EMITTER'S AND OBSERVER'S CLOCKS AND RULERS MUST DIFFER

THE VELOCITY TRIANGLE

Observation & Measurement

- DRAW A CIRCLE ABOUT D=ct_1, OF RADIUS x_1
  - A is Time of Emission of Radar Pulse, Sam's Query
  - B is Time of Receipt of Radar Pulse , and Sally's Reply
  - C is Sally's Location Determined by A and B
    - $x_1=(3-1)/2=1$
    - $ct_1=(3+1)/2=2$
    - $v=x_1/ct_1=0.5c$



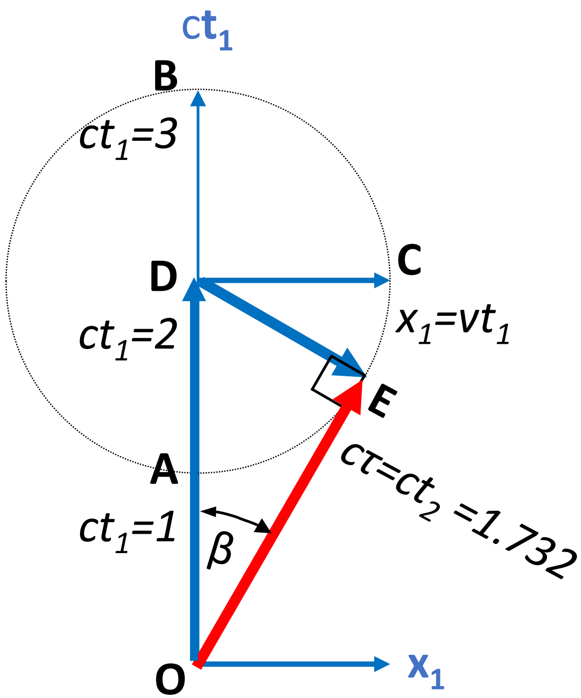
THE VELOCITY TRIANGLE

The Brehme Angle *Beta*

- DRAW LINE FROM O TANGENT TO CIRCLE AT E
- OE IS THE INVARIANT

$$c\tau = ct_2 = \sqrt{(ct_1)^2 - x_1^2} = ct_1\sqrt{1-\left(\frac{v}{c}\right)^2}$$

- VELOCITY TRIANGLE IS ODE
- OE SEPARATED FROM SAM’S TIME AXIS OD BY Brehme Angle *β=sin⁻¹(v/c)*



THE VELOCITY TRIANGLE

Worldline vs. Timeline

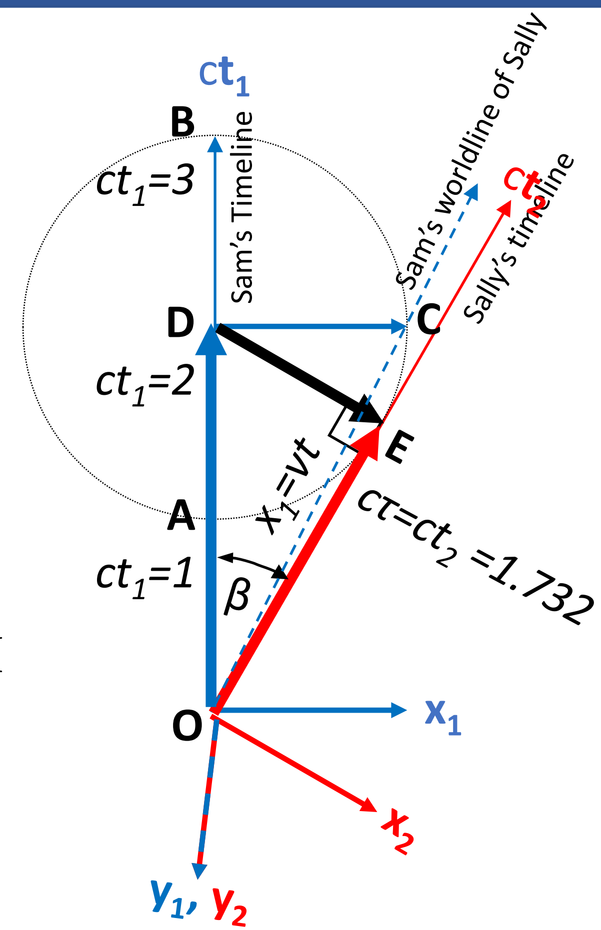
- VELOCITY TRIANGLE ODE:
  - Sally's ct_2 Time Axis through OE
  - Sally's x_2 Spatial Axis Normal to OE, in Direction of Motion

- WORLDLINES AND TIMELINES
  - Minkowski Worldline OC
    - Locus of Successive Measurement of Sally by Sam
  - Timelines OD and OE (NEW)
    - Locus of Proper Clock Ticks from O by Sam and Sally

- TRIGONOMETRIC LORENTZ TRANSFORM

$$x_1 = \frac{x_2 + ct_2 \cdot \sin(\beta)}{\cos(\beta)}$$

$$ct_1 = \frac{ct_2 + x_2 \cdot \sin(\beta)}{\cos(\beta)}$$

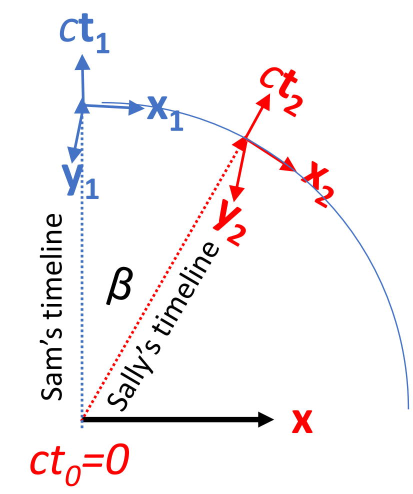


THE VELOCITY TRIANGLE

Rotated Reference Frames Propagating at *c*

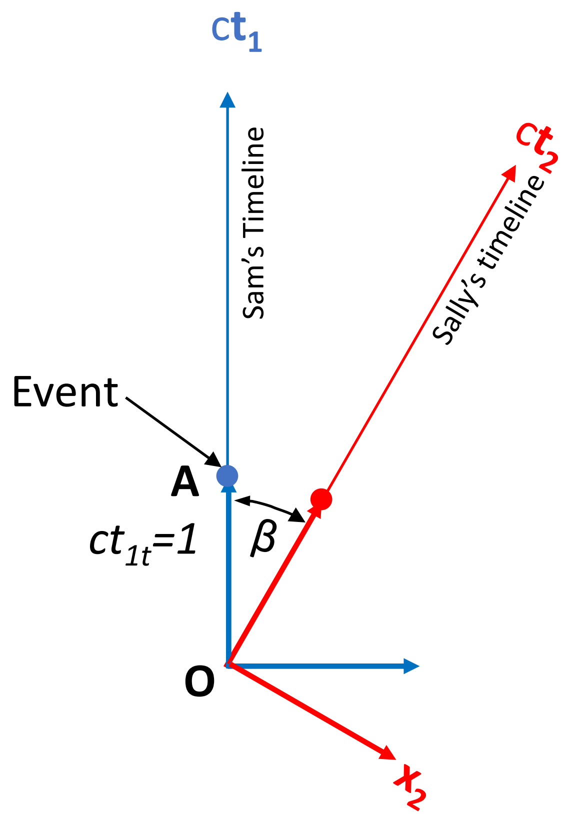
- ALL MATTER PROPAGATES AT PROPER VELOCITY *c* IN A SINGLE DIRECTION
  - That direction defines its time axis unit vector *c***t**
  - Spatial axes' unit vectors **x, y, z** normal to c**t**.

- RELATIVE VELOCITY: PROJECTION OF ONE BODY'S TEMPORAL VELOCITY ONTO TEMPORAL/SPATIAL AXES OF ANOTHER
  - Through β and Lorentz Transform
  - Does Not Exist as an Independent Variable
  - Maximum Relative Velocity *c* at $\beta = 90^o$



LIGHT AND REFERENCE FRAMES

- SAM REQUESTS SALLY'S TIME AT EVENT A=(1,0)



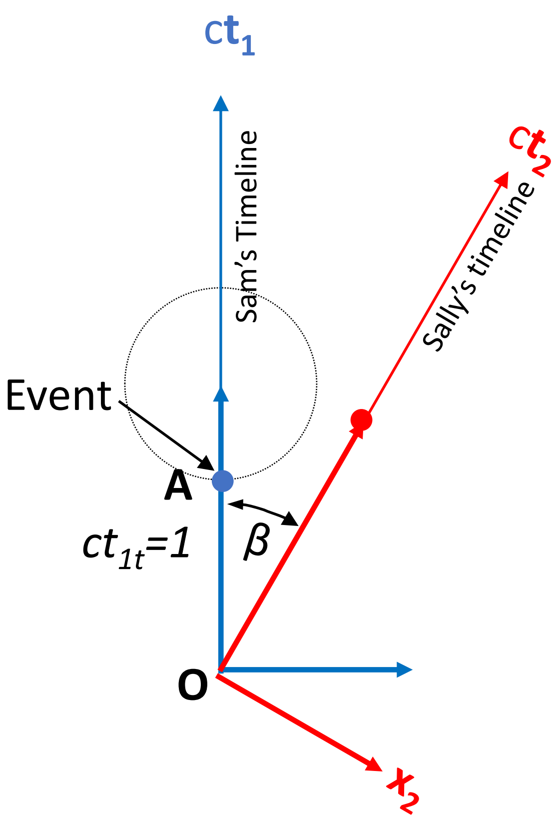
LIGHT AND REFERENCE FRAMES

Tangent to Observer's Timeline, *v=c*

- LIGHT PROPAGATES AS A SPHERE EXPANDING AT *c* IN FOUR-DIMENSIONAL SPACE

$$(c\Delta t)^2 = \Delta x^2 + \Delta y^2 + \Delta z^2$$

- ANCHORED AT POINT OF EMISSION A
- CENTERED ON EMITTER



LIGHT AND REFERENCE FRAMES

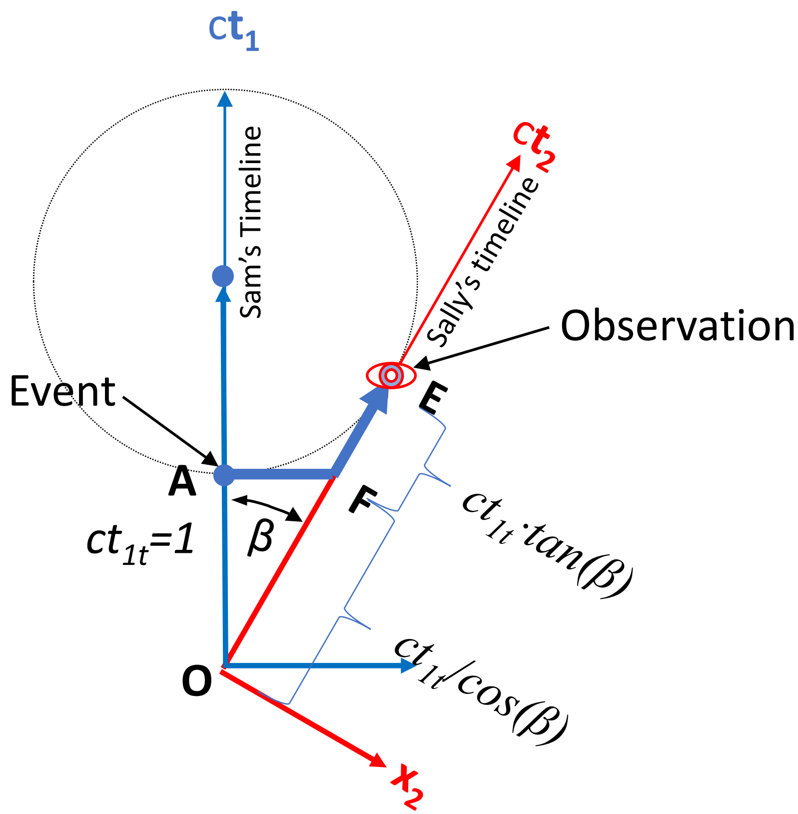
Tangent to Observer's Timeline, *v=c*

- EVENT OBSERVED AT E:
  - Light Sphere Tangent to Sally's TimeLine
  - Radial Velocity of *c*

- TIME INTERVAL OA DOPPLER-SHIFTED ALONG AFE TO TIME OF RECEIPT OE

$$ct_{2r} = ct_{1t}\,\frac{1+\sin(\beta)}{\cos(\beta)}$$

Equal units of space in the reference frame of origination, to equal units of time in reference frame of receipt.

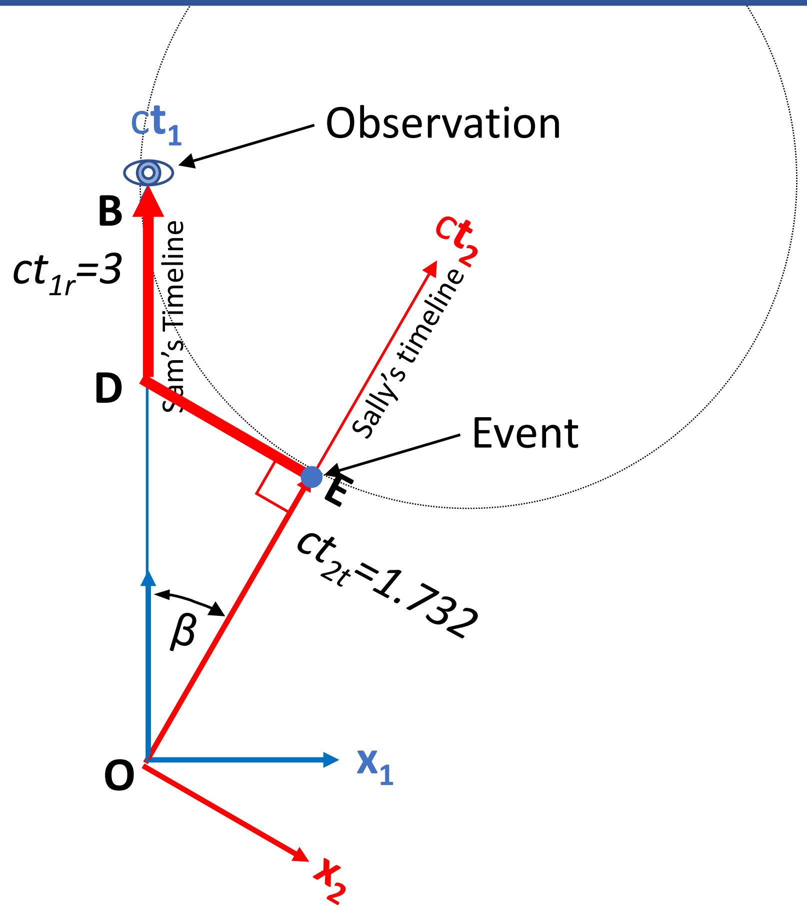


LIGHT AND REFERENCE FRAMES

Tangent to Observer's Timeline, *v=c*

- SALLY'S REPLY IS EVENT E, SIMULTANEOUS WITH HER RECEPTION OF A
  - Tangent to Sam's Timeline at B
  - Doppler-Shifted Along Path EDB to Sam's Time of Observation at B (3, 0):

$$ct_{1r} = ct_{2t}\,\frac{1+\sin(\beta)}{\cos(\beta)} = 3$$



LIGHT AND REFERENCE FRAMES

Sam Determines Sally's Location "at" C

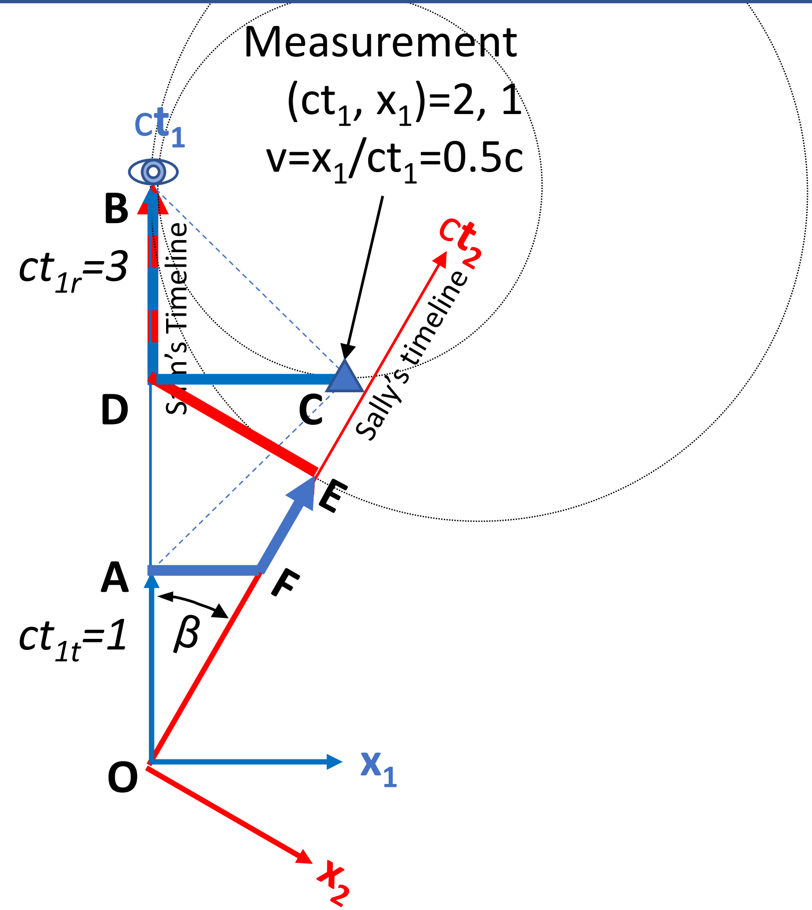
- SAM LOCATES SALLY AT C:

$$x_1 = \frac{ct_{1r} - ct_{1t}}{2} = 1$$

$$ct_1 = \frac{ct_{1r} + ct_{1t}}{2} = 2$$

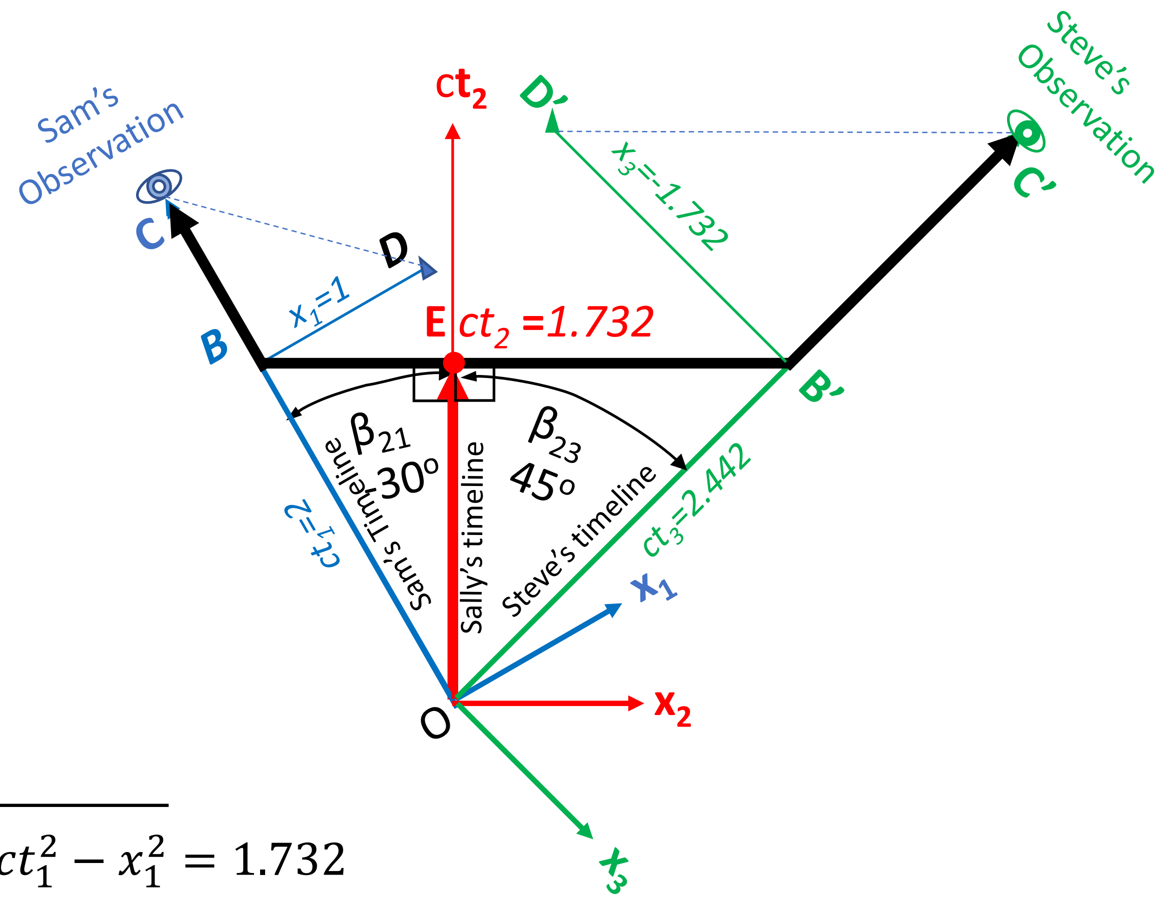
$$v = \frac{x_1}{ct_1} = 0.5$$

- LIGHT FROM C SIMULTANEOUS AT B WITH LIGHT FROM F:
  - Sally is "at" C.
  - Different Doppler Shifts
    - CDB=1.0
    - EDB=1.732

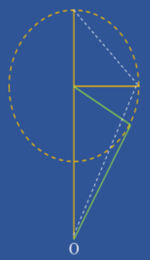


MULTIPLE REFERENCE FRAMES

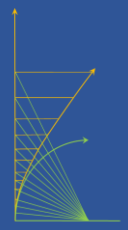
All Reference Frames Agree on Proper Time of the Event



$$c\tau = \sqrt{ct_3^2 - x_3^2} = \sqrt{ct_1^2 - x_1^2} = 1.732$$



SUMMARY



- VELOCITY TRIANGLE
  - Separates Event, Observation and Measurement
  - Eliminates "Rubber Rulers" and "Asynchronous Clocks"
  - The Measurement of Both Clock and Rulers Affected by Relativistic Doppler Shift
- PROVIDES MORE CLEAR, LESS MYSTERIOUS DEMONSTRATION OF ALL ASPECTS OF SPECIAL THEORY USING SIMPLE GEOMETRY
  - Additive Velocities
  - Mass, Momentum and Energy
- CAN BE APPLIED TO ACCELERATED (NON-INERTIAL) SYSTEMS