Chesapeake Section of the American Association of Physics Teachers

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PULLING A SPOOL

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A spool of ribbon is on a horizontal table.
The ribbon is pulled at an angle relative to the table.
It is pulled gently enough that the spool rolls without slipping.

VOTE! Raise your hand if you think the spool will:

(a) roll forward

(b) roll backward

(c) it depends

Let’s try the demo and see!

So the direction of rolling
depends on the angle of pulling:

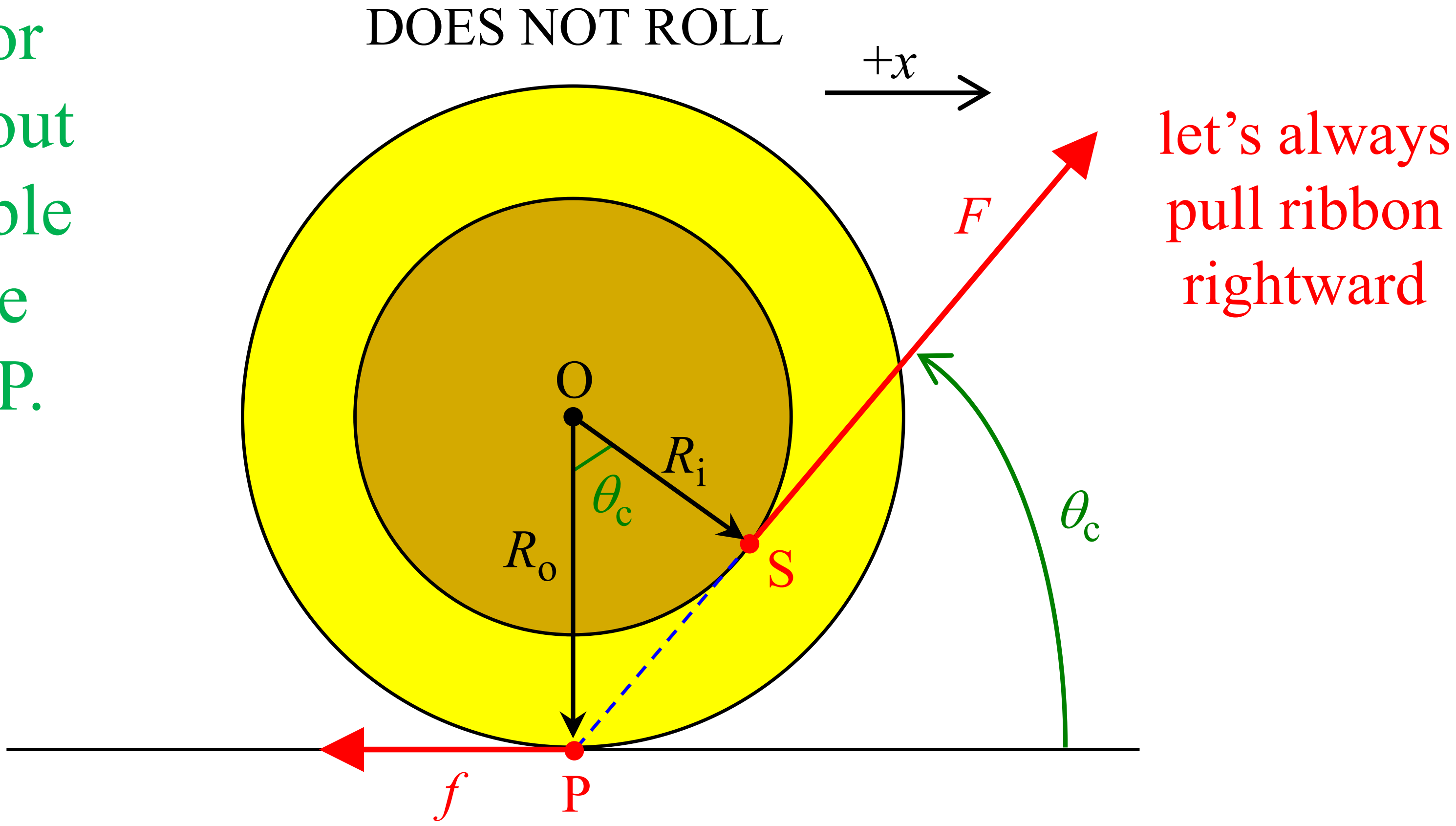
At near-horizontal angles the spool rolls FORWARD
but at near-vertical angles it rolls BACKWARD.

If so, there must be some intermediate angle
at which it *cannot* roll without slipping.
Call that the critical pulling angle θ_c.
What determines its value?

For a cylindrically symmetric spool, θ_c is determined entirely by its inner radius R_i and outer radius R_o:

Neither the pulling force F nor the friction f exert a torque about the contact point P with the table and so the spool cannot rotate forward or backward around P.

$$\theta_c = \cos^{-1}(R_i/R_o)$$

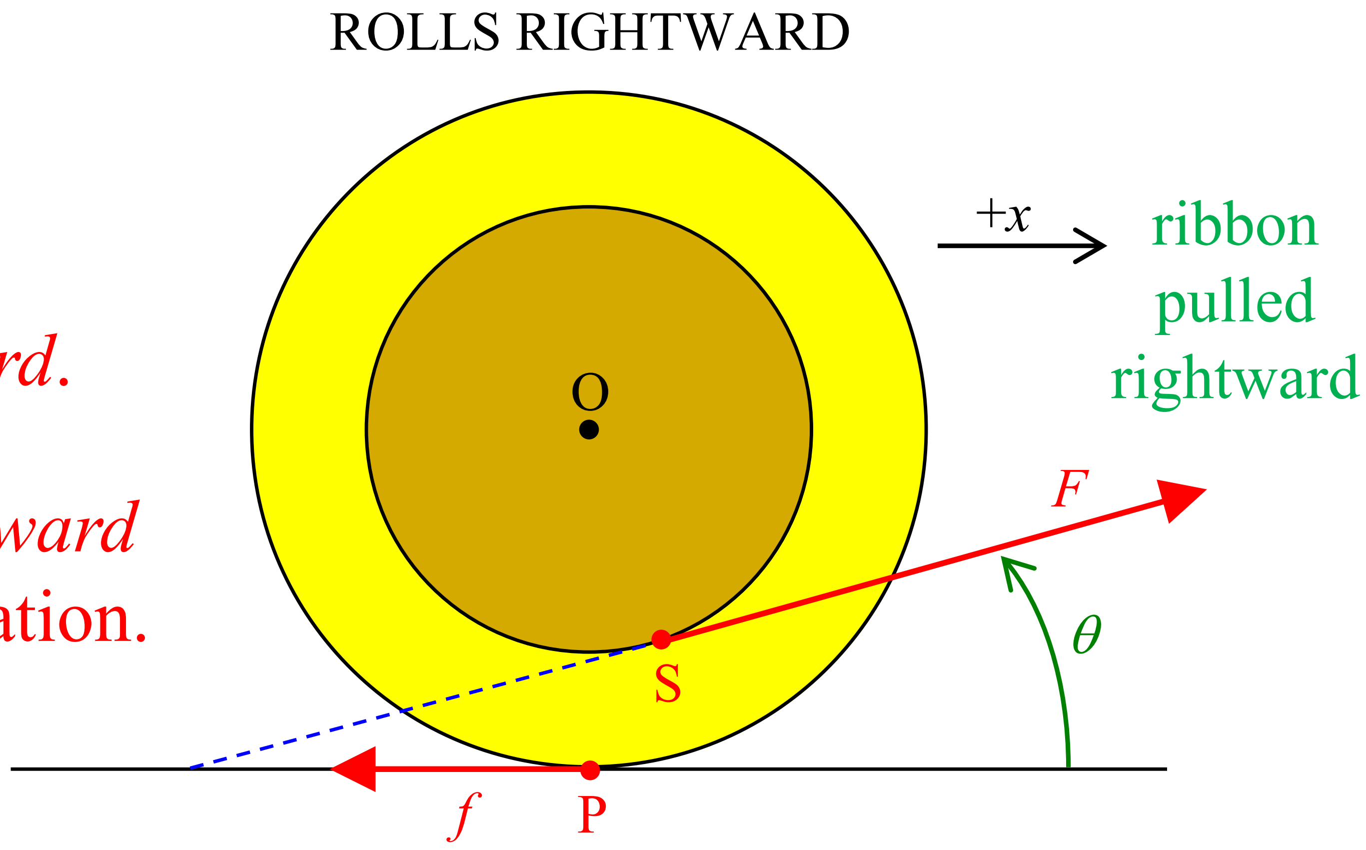


There are 4 possible ranges for the pulling angle to get rolling without slipping.

The first range is $0° \leq \theta < \theta_c$:

Only F has a torque about P.
That proves the spool rolls *forward*.

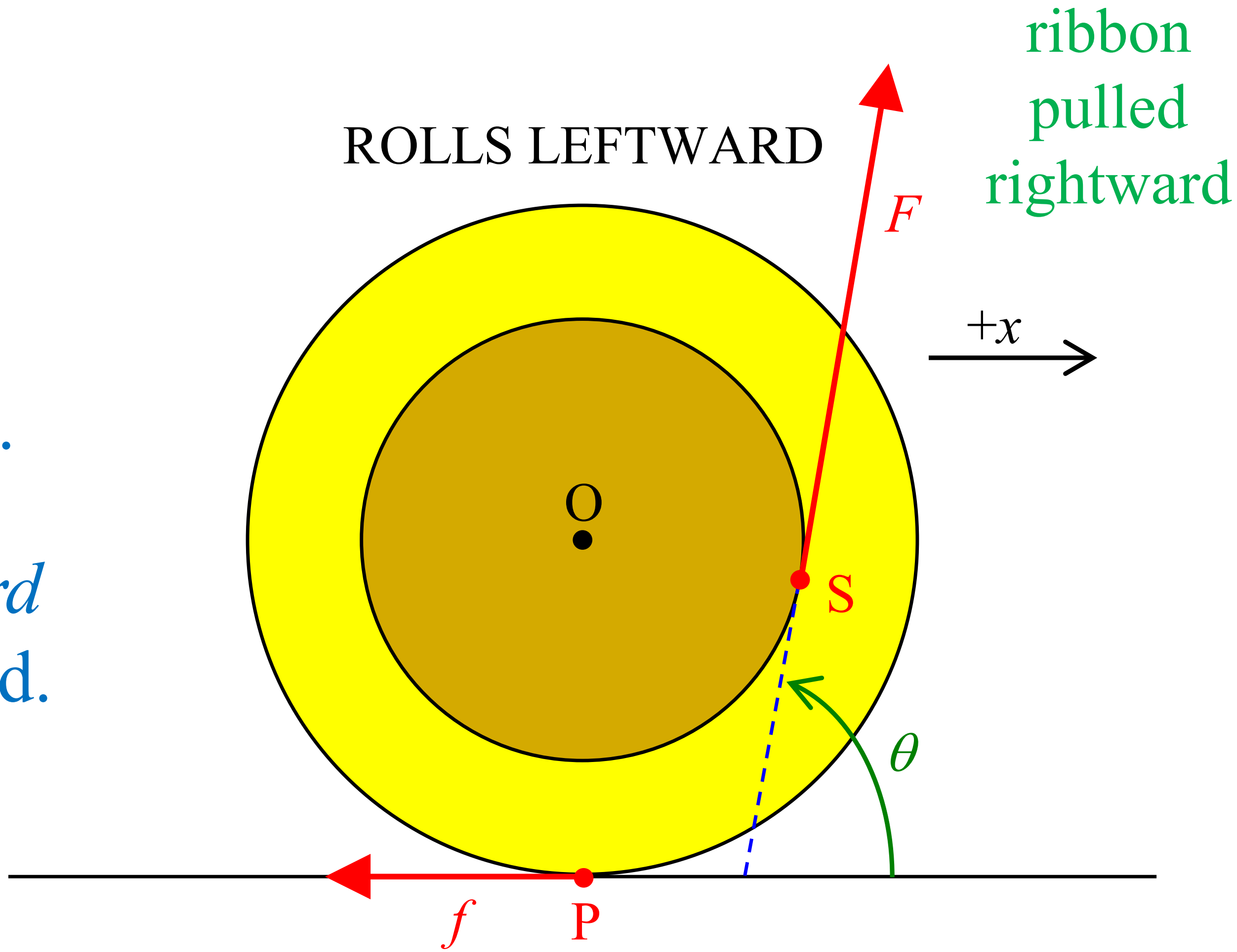
Then the friction f must point *backward*
to give a clockwise angular acceleration.



The second range is $\theta_c < \theta < 90°$:

Only *F* has a torque about P.
That implies the spool rolls *backward*.

Then the friction *f* must point *backward* to cause the spool to translate backward.



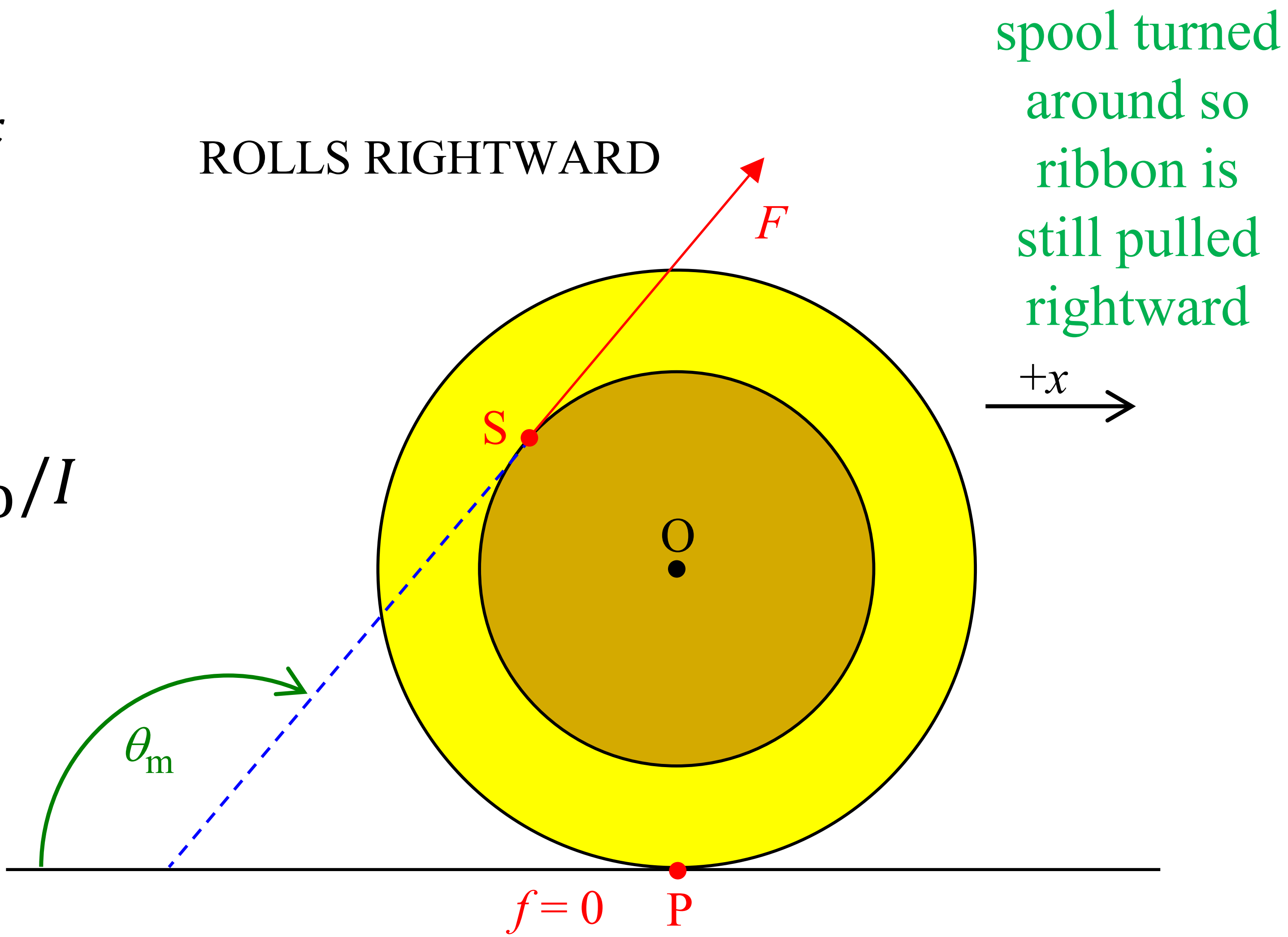
There *may* be a second special angle:
a maximum angle $\theta_m > 90°$
at which the friction force becomes zero!

N2L for force: $-F\cos\theta_m = Ma_x$

N2L for torque: $FR_i = I\dfrac{a_x}{R_o}$

eliminate F: $-\cos\theta_m = MR_iR_o/I$

θ_m exists iff $I \geq MR_iR_o$



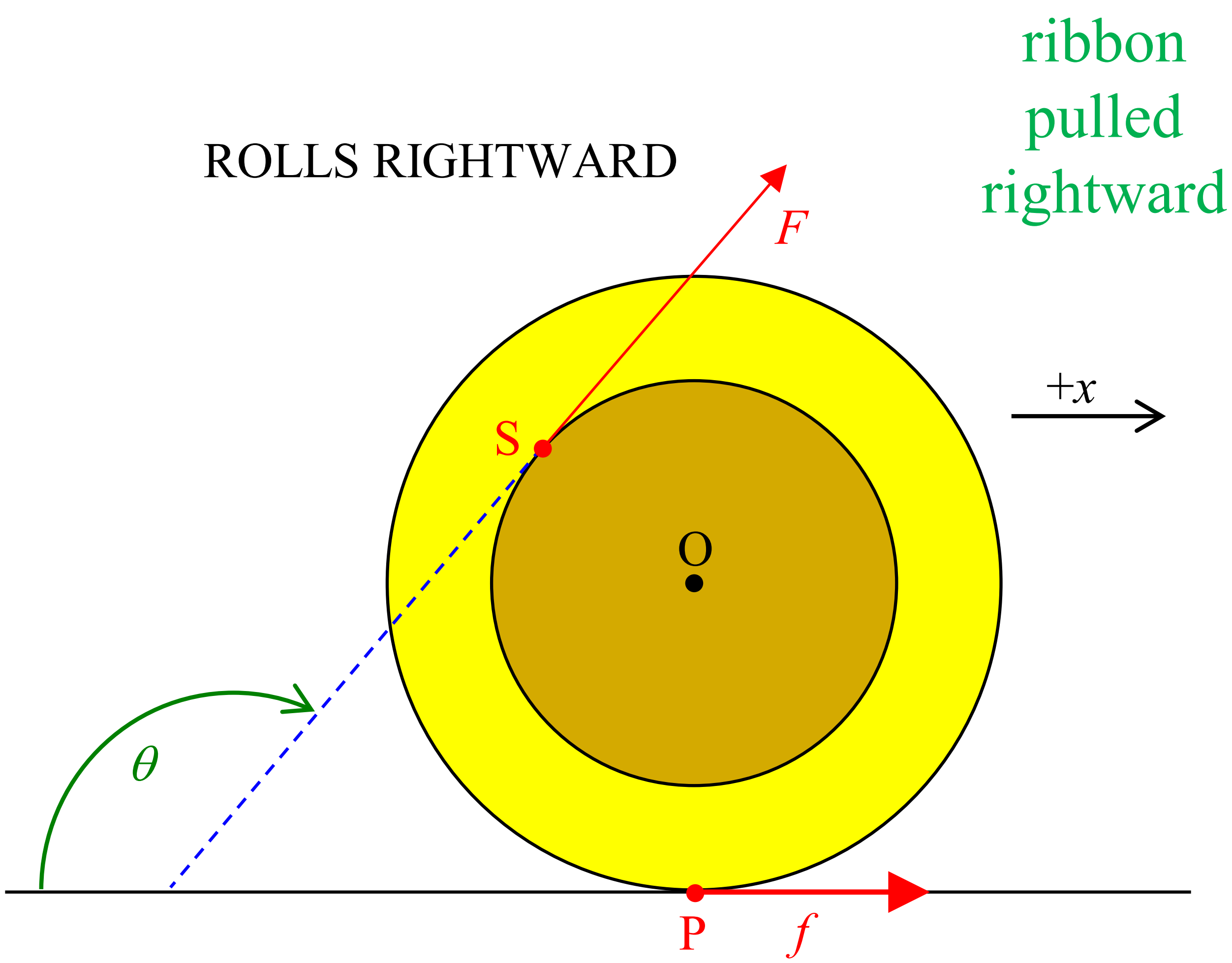
The third range is $90° < \theta < \theta_m$ (if θ_m exists) or $90° < \theta \leq 180°$ (if not):

Only F has a torque about P. That proves the spool rolls *forward*.

Eliminate a_x between N2L for force and for torque to get

$$f = F\frac{\cos\theta - \cos\theta_{\mathrm{m}}}{1 + MR_{\mathrm{o}}^2/I}$$

so that the friction f points *forward*.



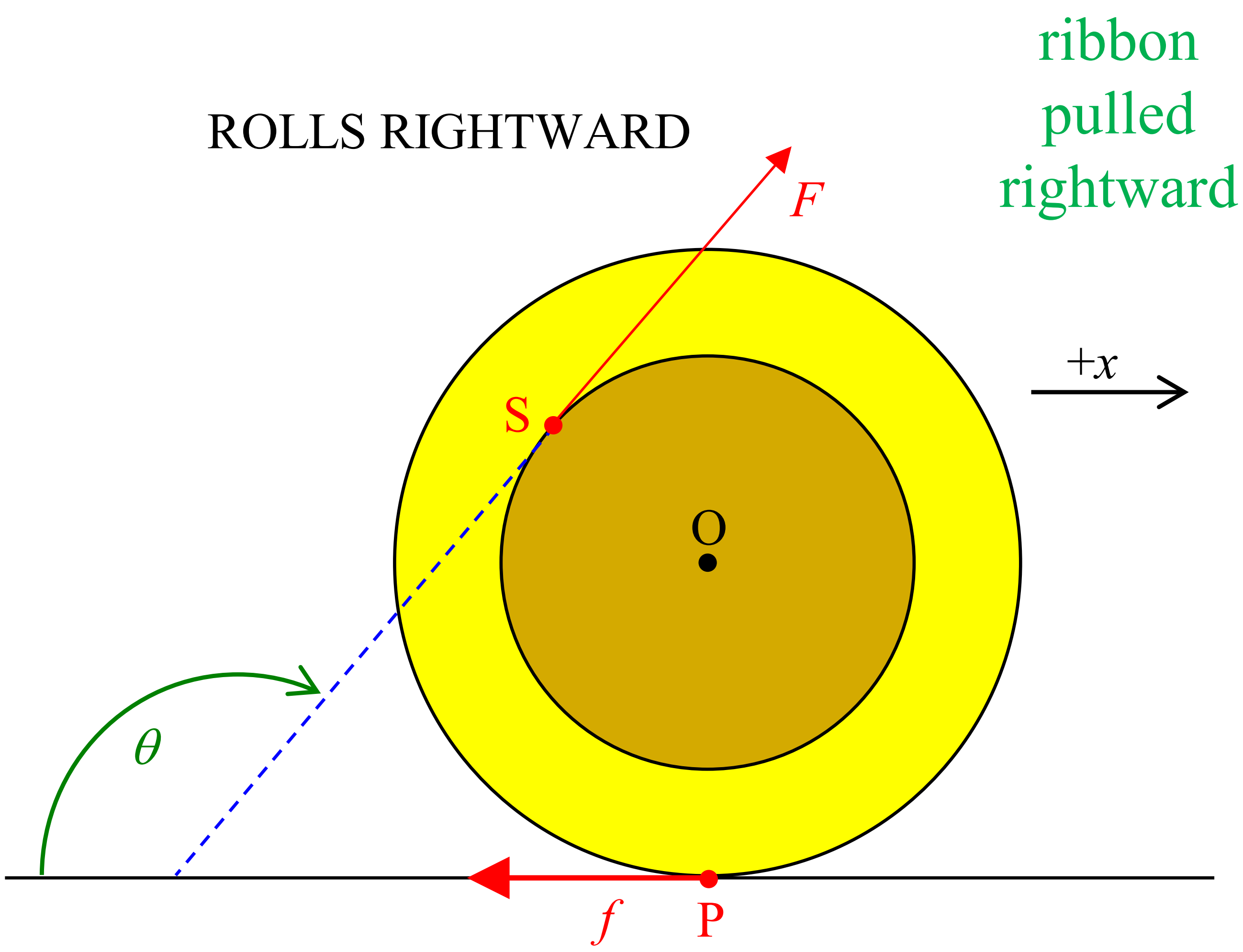
The fourth and final range is
$\theta_m < \theta \leq 180°$ (if θ_m exists):

Only F has a torque about P.
So again the spool rolls *forward*.

But now

$$f = F\frac{\cos\theta_{\mathrm{m}} - \cos\theta}{1 + MR_0^2/I}$$

which this time shows the friction f points *backward*.



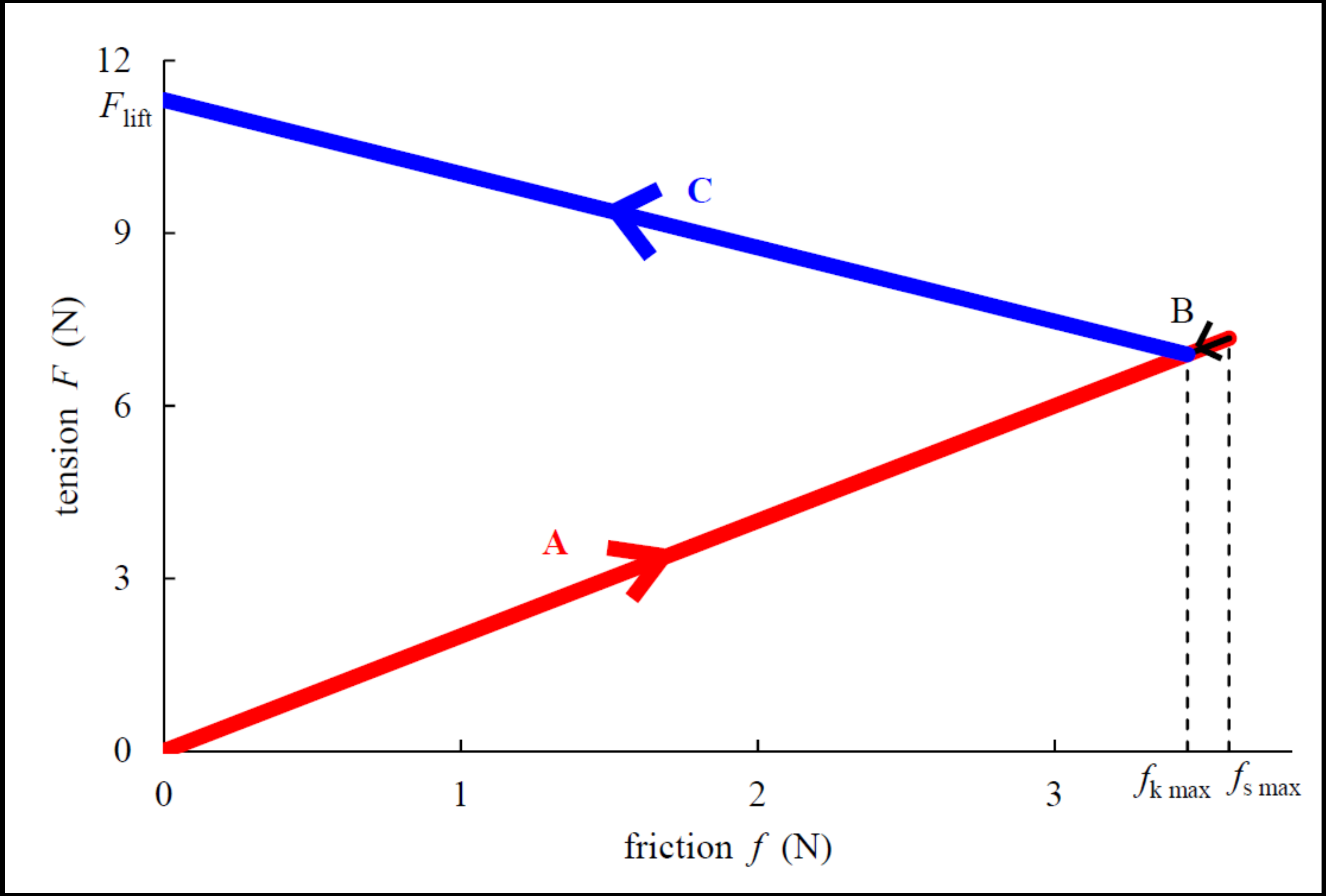
SUMMARY TABLE
(assuming ribbon pulling *defines* the forward direction)

Angular Range	Direction of Rolling	Direction of Friction
$0° \leq \theta < \theta_c$	forward	backward
$\theta_c < \theta < 90°$	backward	backward
$90° < \theta < \theta_m$	forward	forward
$\theta_m < \theta \leq 180°$	forward	backward

All of these directions can be obtained by quick qualitative arguments *except* for friction at angles beyond 90° which requires a formal N2L analysis.

Friction need not be opposite the direction of rolling!
Nor need it be opposite the direction of pulling!

Pull at a critical angle of 60°:



Starting from the origin, static friction prevents the spool from moving and we rise up along line A until the maximum value of the static friction $f_{\text{s max}}$ is attained. Since $\mu_k < \mu_s$ we must reduce the applied force by backing up along line B once slipping starts. At the maximum value of the kinetic friction $f_{\text{k max}}$ the spool slips in place. From that point, if we now increase the pulling force, we will progressively reduce the normal force and hence the kinetic friction along line C until we lift the spool off the table once the friction and normal forces fall to zero.

QUESTIONS?

Comments welcome by email to mungan@usna.edu.

Visit my webpage at usna.edu/Users/physics/mungan.